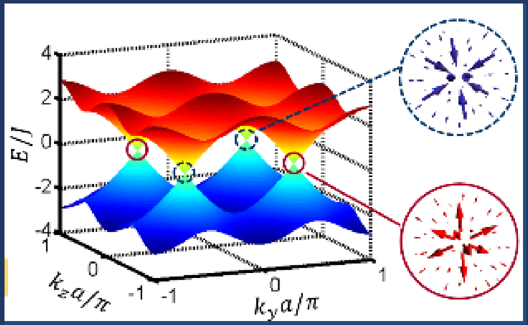
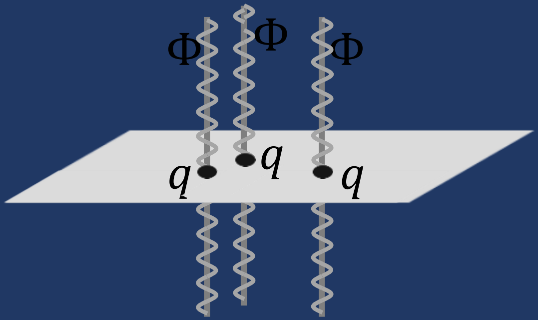
Engineering Synthetic Gauge Fields, Weyl Semimetals, and Anyons





T. Dubček[1], M. Todorić[1], B. Klajn[1], C. J. Kennedy[2], L. Lu[2], R. Pezer[5], D. Radić[1], D. Jukić[4], W. Ketterle[2], M. Soljačić[2], <u>H. Buljan[1]</u>

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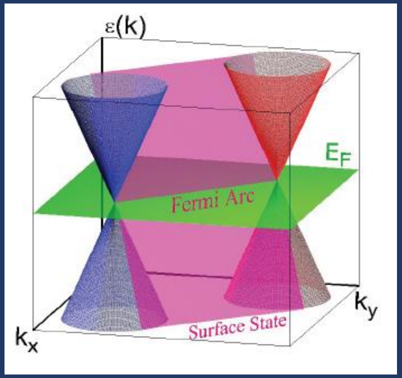
Sveučilište u Zagrebu

PMF

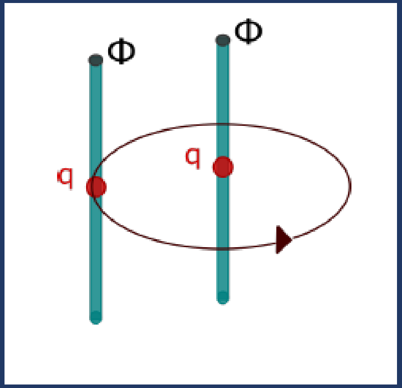
Funding:
QuantiX Center of Excellence
HRZZ - SynthMagIA

Overview

- Introduction: synthetic gauge/magnetic fields for atoms
- *Weyl points in 3D optical lattices: Synthetic Magnetic Monopoles in Momentum Space* by Tena Dubček, Colin J. Kennedy, Ling Lu, Wolfgang Ketterle, Marin Soljačić, & **H.B.** Phys. Rev. Lett. 114, 225301 (2015)



- *The Quantum Hall Effect with Wilczek's charged magnetic flux tubes instead of electrons* by Marija Todorić, Dario Jukić, Danko Radić, Marin Soljačić, H.B., in preparation



- *Quasimomentum distribution and expansion of an anyonic gas* by Tena Dubček, Bruno Klajn, Robert Pezer, H.B., and Dario Jukić, arXiv:1707.04712 [cond-mat.quant-gas]
- Outlook and Conclusion

Ultracold atomic gases
potential quantum emulators

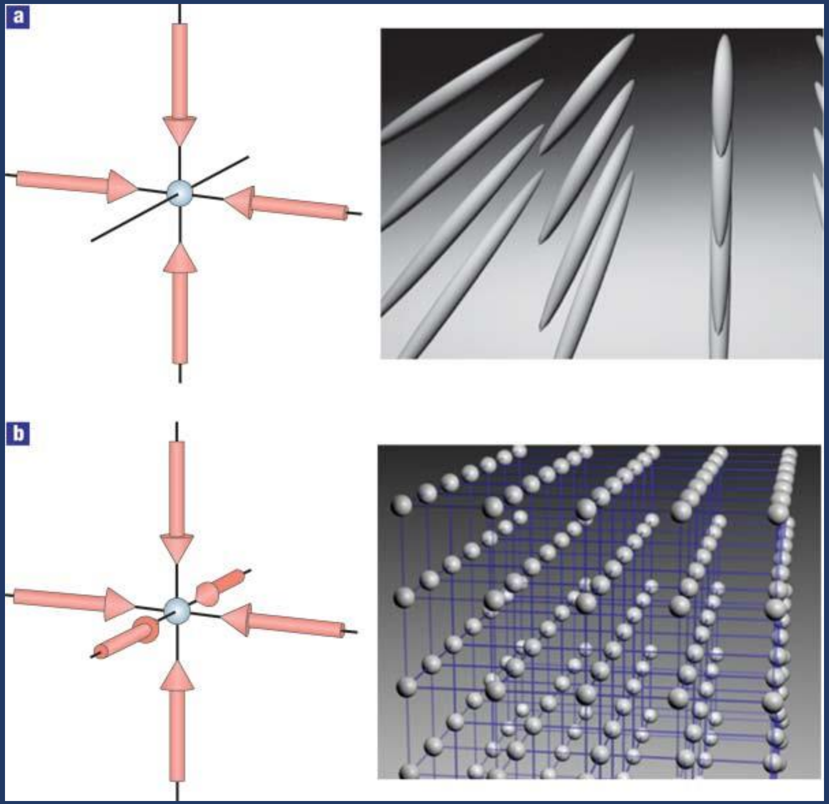
1. External potentials can be changed
   - harmonic potential, optical lattices,
   - honeycomb (graphene),
   - random (Anderson localization),
   - artificial gauge fields

2. Interactions can be tuned –
   - Feshbach resonances,
   - changing hopping in lattices, ...

3. DIMENSIONALITY – 1D, 2D, 3D

4. Weak coupling to ENVIRONMENT

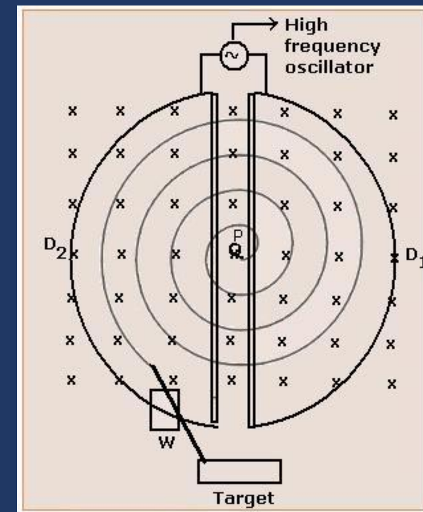
Isolated many body systems
out-of-equilibrium !!!



However, atoms are neutral.
Desirable to have magnetic effects,
Hall effect, topological phases …

e.g., see Bloch, Dalibard, Nascimbene, Nature Physics **8**, 267 (2012).

Charged particles in magnetic fields



CLASSICAL SYSTEMS

Newton's 2nd law:

$$m\frac{d^2\mathbf{r}}{dt^2} = \mathbf{F} = q\mathbf{v}\times\mathbf{B}$$

Manifestation (examples)
- mass spectrometer
- Cyclotron orbits

QUANTUM SYSTEMS

Schrodinger equation:

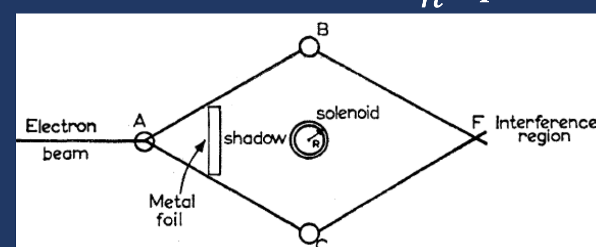
$$\mathbf{p} \to \mathbf{p} - q\mathbf{A}$$

$$\mathbf{B} = \nabla\times\mathbf{A}$$

$$i\frac{\partial\psi}{\partial t} = \frac{1}{2m}(\mathbf{p} - q\mathbf{A})^2\psi + V\psi$$

Manifestation (examples)
- Landau levels
- quantum Hall effect

Aharonov-Bohm phase: $\varphi = \frac{q}{\hbar}\oint_P \vec{A}\cdot d\vec{l} = \frac{q}{\hbar}\phi_B$



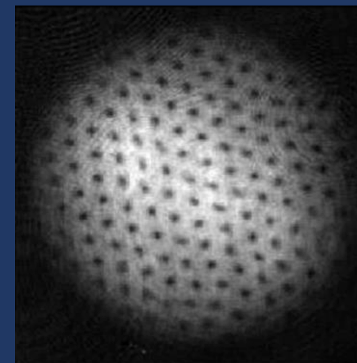
Significance of Electromagnetic Potentials in the Quantum Theory

Y. Aharonov and D. Bohm
H. H. Wills Physics Laboratory, University of Bristol, Bristol, England

(Received May 28, 1959; revised manuscript received June 16, 1959)

Synthetic magnetic fields for atoms - methods

Existing methods:



J.R.Abo-Shaer *et al.*, Science **292**, 476 (2001).

- *Rotation of the gas*

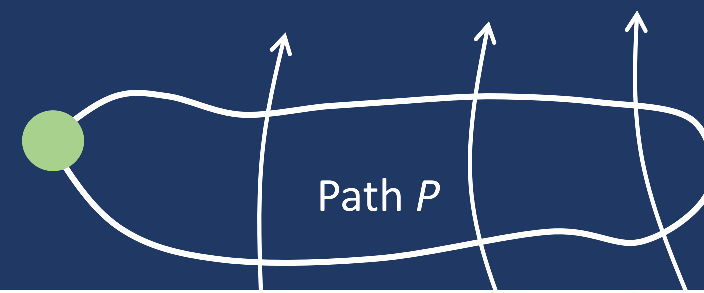
  [Lorentz force] ⟷ [Coriolis force]

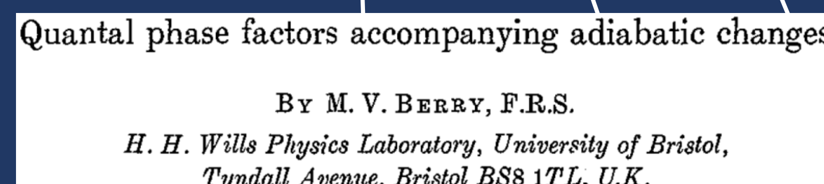
- *Laser-atom interactions*

  [Aharonov-Bohm phase] ⟷ [Berry phase]

$$\varphi = \frac{q}{\hbar}\oint_P \vec{A}\cdot d\vec{l} = \frac{q}{\hbar}\phi_B$$

Lin *et al.*, Nature **462**, 628 (2009).





Quantal phase factors accompanying adiabatic changes

By M. V. Berry, F.R.S.

H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, U.K.

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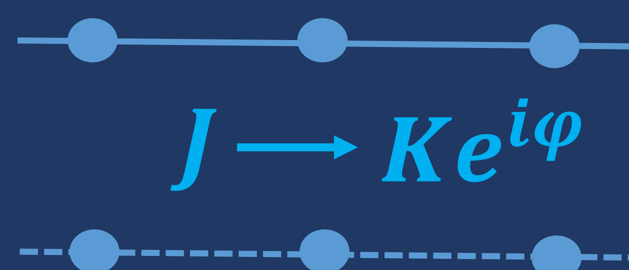
* *Optical lattices* (modulation tehniques, laser assisted tunneling, …)

Struck et al. PRL 108, 225304 (2012)

Aidelsburger,…, Bloch, PRL 111, 185301 (2013)

Myake,…, Ketterle, PRL 111, 185302 (2013)



$$J \longrightarrow K e^{i\varphi}$$

Synthetic gauge fields in *neutral atomic quantum gases* via laser-atom interactions

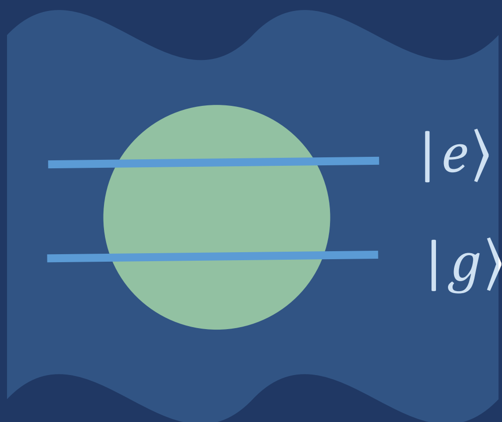
Dalibard et al. Rev. Mod. Phys. (2011).

$$H = \left(\frac{P^2}{2M} + V\right)\hat{1} + U,$$

Center of mass

$$U = \frac{\hbar\Omega}{2}\begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{pmatrix}.$$

Light atom coupling



Dressed states:

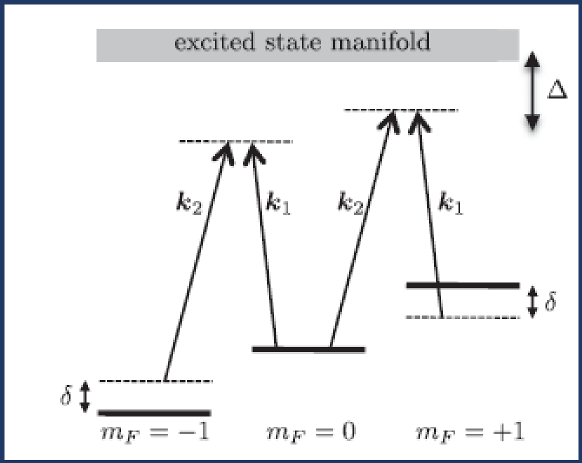
$$|\chi_1\rangle = \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{pmatrix}, \quad |\chi_2\rangle = \begin{pmatrix} -e^{-i\phi}\sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix},$$

For atom in $|\chi_1\rangle$ that adiabatically moves in the light field:

$$i\hbar\frac{\partial\psi_1}{\partial t} = \left[\frac{(P-A)^2}{2M} + V + \frac{\hbar\Omega}{2} + W\right]\psi_1.$$

$$A(r) = i\hbar\langle\chi_1|\nabla\chi_1\rangle = \frac{\hbar}{2}(\cos\theta - 1)\nabla\phi.$$

Population of excited states should be avoided!!!
Spontaneous emission heats the gas

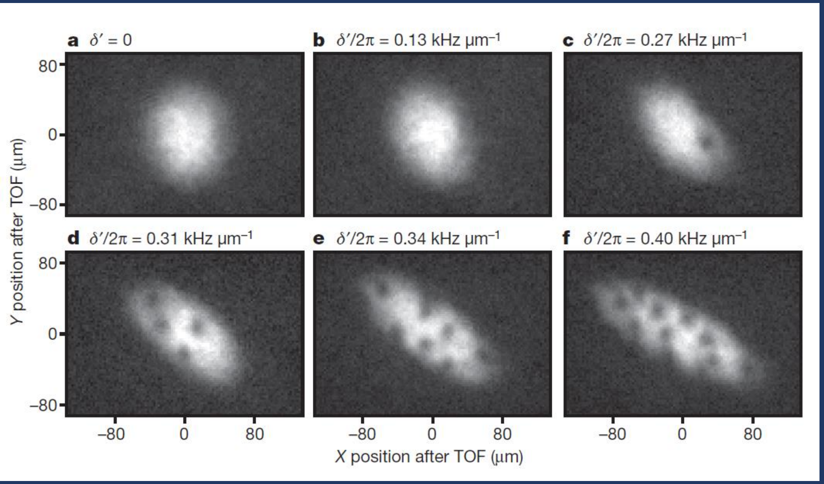


Lin *et al.*, Nature **462**, 628 (2009).

Signatures

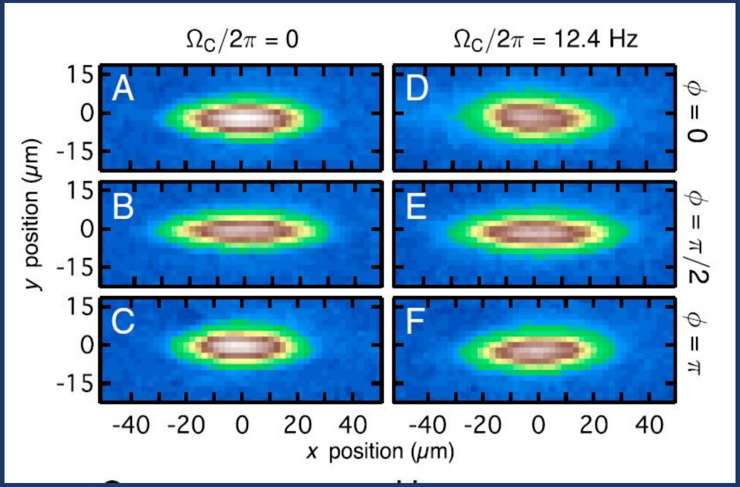
Synthetic gauge/magnetic fields in *neutral atomic quantum gases* via laser-atom interactions (ctd.)

Signature: Formation of vortices in a BEC



Lin … Spielman, Nature **462**, 628 (2009).

Hall effect, signature skewness of the cloud



Le Blanc *et al.*, PNAS, (2012).

Weyl points

PRL **114,** 225301 (2015) PHYSICAL REVIEW LETTERS week ending 5 JUNE 2015

Weyl Points in Three-Dimensional Optical Lattices: Synthetic Magnetic Monopoles in Momentum Space

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[2]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

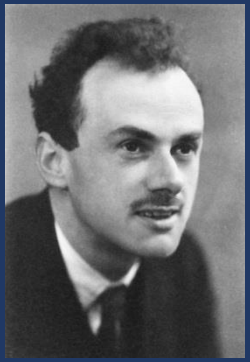
(Received 24 December 2014; published 3 June 2015)

Weyl fermions

Relativistic quantum field theory: DIRAC, MAJORANA, WEYL FERMIONS

DIRAC fermions

- electron, muon, …
- mass
- Dirac equation



Paul Dirac

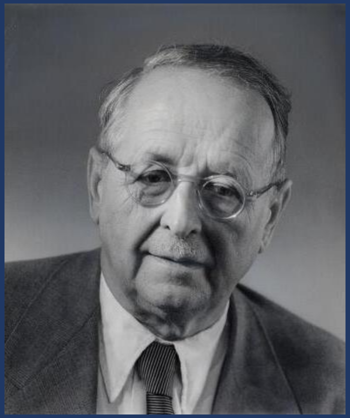
MAJORANA fermions

- not observed in particle physics
- particle is its own antiparticle
- Today: neutrinos?

Ettore Majorana

WEYL fermions

- not observed in particle physics
- mass = 0
- neutrinos – believed to be Weyl fermions until neutrino oscillations were observed

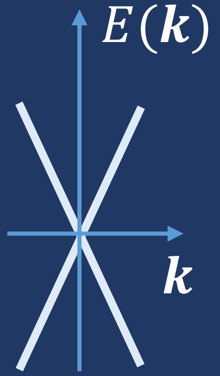


Hermann Weyl

WEYL Hamiltonian

$$H = \hbar v \boldsymbol{\sigma} \cdot \boldsymbol{k}$$

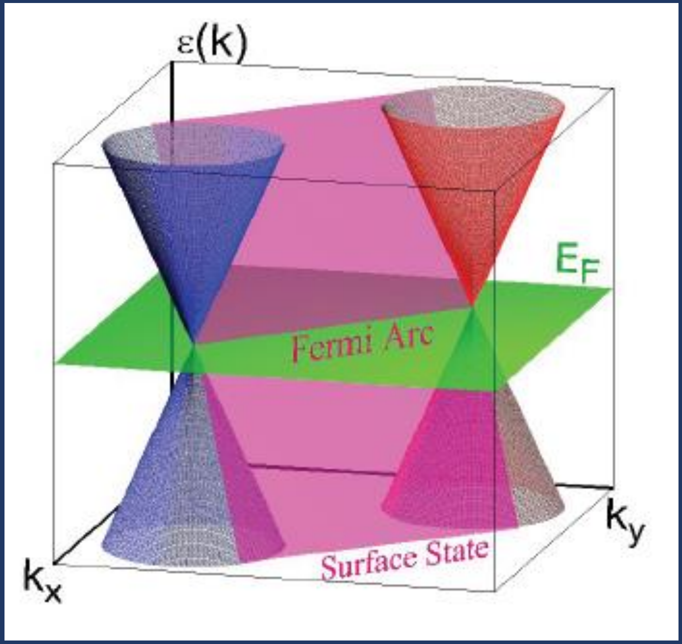
chirality



3D

Weyl semimetals

- Conduction and valence band touch at Weyl points
- Energy vs. $\boldsymbol{k}$ linear along all three dimensions – massless fermions
- Low energy electrons described by the Weyl Hamiltonian $H = \hbar v \boldsymbol{\sigma} \cdot \boldsymbol{k}$
- Time reversal symmetry or/and inversion symmetry must be broken in these materials
- Robust – Weyl points of different chirality can only be annihilated
- Fermi arc surface states



Review: A. M. Turner and A. Vishwanath, arXiv:1301.0330.

ELUSIVE, only recently observed in condensed matter:

S.-Y. Xu et al., Science 349, 613 (2015).

B. Q. Lv et al., Phys. Rev. X 5, 031013 (2015)

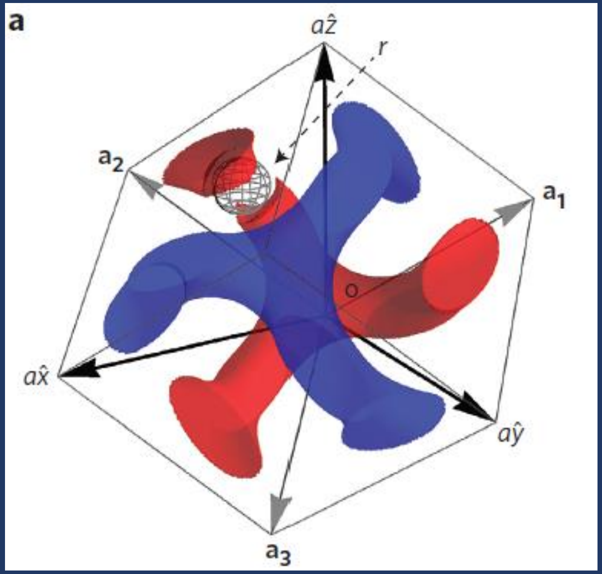
3D

Weyl points in photonics

- Theoretically proposed in double giroid photonic structures
- Inversion symmetry breaking – structural design
- Time reversal symmetry breaking – gyroelectric materials

$$\varepsilon(|\mathbf{B}|) = \begin{pmatrix} \varepsilon_{11}(|\mathbf{B}|) & i\varepsilon_{12}(|\mathbf{B}|) & 0 \\ -i\varepsilon_{12}(|\mathbf{B}|) & \varepsilon_{11}(|\mathbf{B}|) & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$$

ELUSIVE, only recently observed in photonics:
L. Lu, Z. Wang, D. Ye, L. Ran, L. Fu, J.D. Joannopoulos, M. Soljačić , Science 349, 622 (2015).



Theory:

L. Lu, L. Fu, J. D. Joannopoulos, and M. Soljačić, Nat. Photonics 7, 294 (2013).

J. Bravo-Abad, L. Lu, L. Fu, H.B., M. Soljačić, 2D Mater. 2 (2015) 034013 (all dielectric superlattices)

Weyl points in momentum space of 3D optical lattices

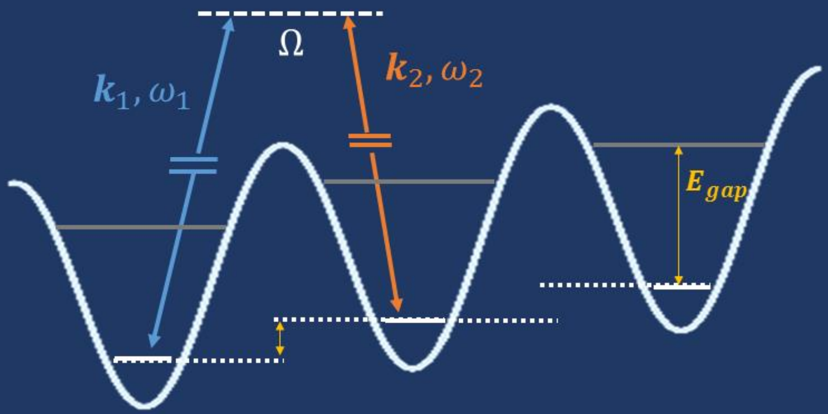
1. Ultracold atomic gases in 3D optical lattices – highly controllable systems
2. Synthetic magnetic fields – can be used to break time reversal and/or inversion symmetry in simple cubic lattice geometry

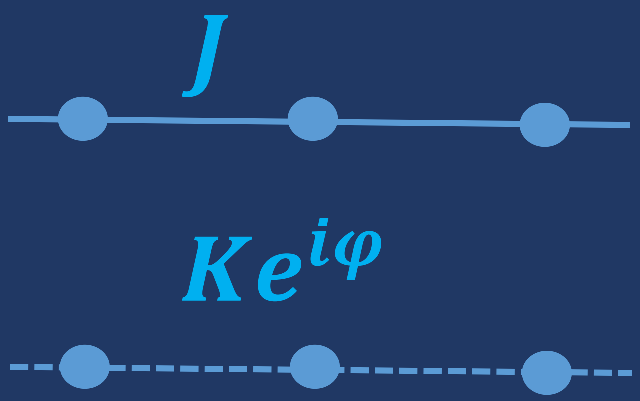
Theoretical work on Weyl pts.:

- Lan, Goldman, Bermudez, Lu, Öhberg, PRB 84, 165115 (2011).
- Jiang, PRAA 85, 033640 (2012).
- Ganeshan, Das Sarma, PRB 91, 125438 (2015).

Weyl points: within experimental reach in systems that realized the Harper-Hofstadter Hamiltonian

- Miyake, Siviloglou, Kennedy, Burton, Ketterle, PRL **111**, 185302 (2013).
- Aidelsburger, Atala, Lohse, Barreiro, Paredes, Bloch, PRL **111**, 185301 (2013).





The α=1/2 Harper-Hofstadter Hamiltonian

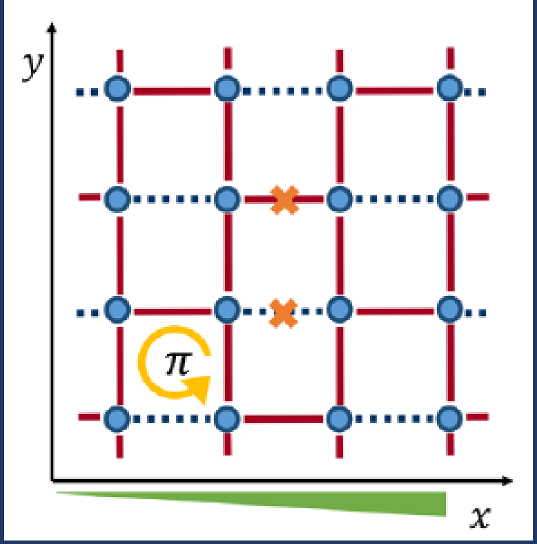
Miyake, Siviloglou, Kennedy, Burton, Ketterle, *Phys. Rev. Lett.* **111**, 185302 (2013)

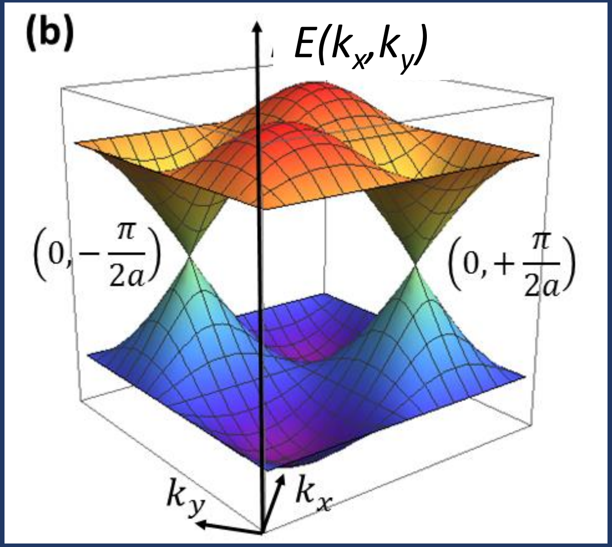
Engineering both the phase and the amplitude of the tunneling matrix elements in 2D optical lattice

$$H_{\alpha=1/2}(\mathbf{k}) = -2\{J_y \cos(k_y a)\sigma_x + K_x \sin(k_x a)\sigma_y\}, \quad + f(k_z)\sigma_z \quad ?$$

$\boxed{0}$ π

$$E_{\alpha=1/2} = \pm 2\sqrt{K_x^2 \sin^2(k_x a) + J_y^2 \cos^2(k_y a)},$$





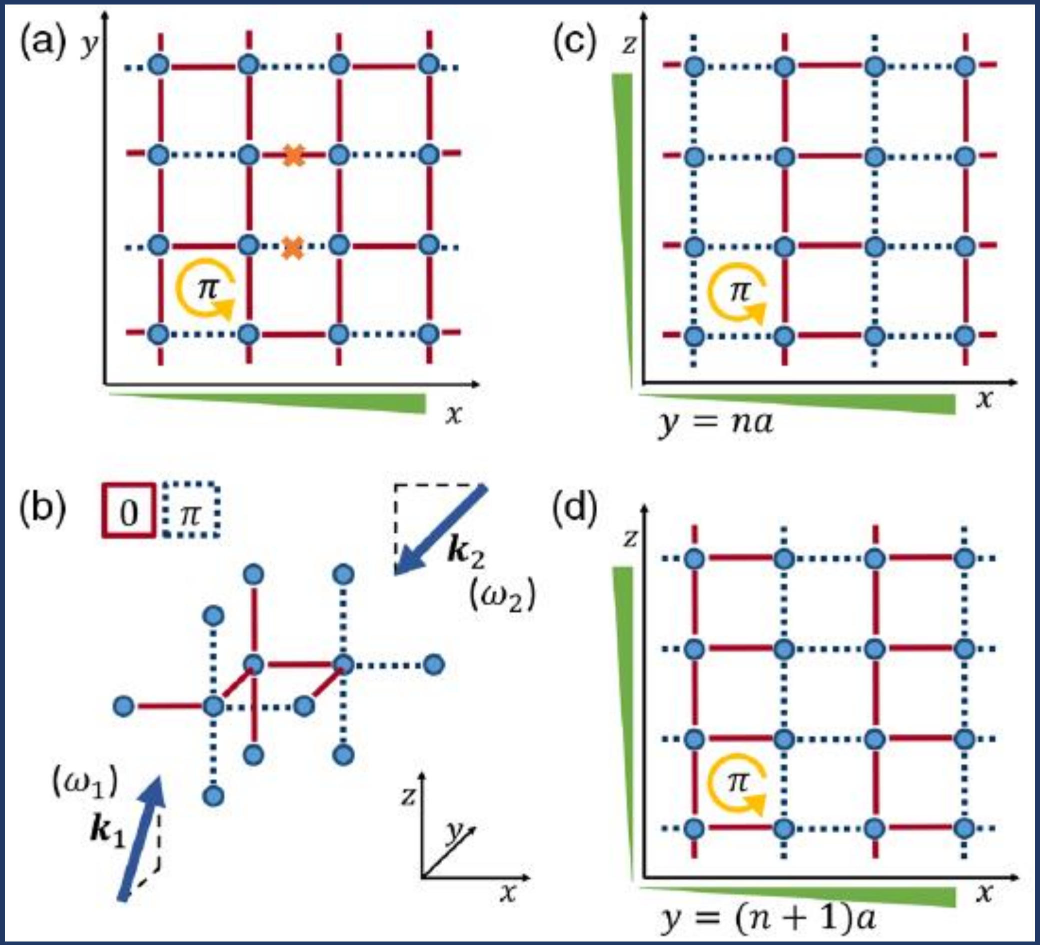
The α=1/2 2D lattice:

- time reversal symmetry
- inversion symmetry
- Dirac points in k-space

Weyl Hamiltonian with laser-assisted tunneling

Laser-assisted tunneling along both x and z directions

$$H_{3\mathrm{D}} = -\sum_{m,n,l}\left(K_x e^{-i\Phi_{m,n,l}} a^\dagger_{m+1,n,l} a_{m,n,l} + J_y a^\dagger_{m,n+1,l} a_{m,n,l} + K_z e^{-i\Phi_{m,n,l}} a^\dagger_{m,n,l+1} a_{m,n,l} + \mathrm{H.c.}\right).$$



$$\Phi_{m,n,l} = \delta\boldsymbol{k} \cdot \boldsymbol{R}_{m,n,l} \equiv m\Phi_x + n\Phi_y + l\Phi_z,$$

$$\left(\Phi_x, \Phi_y, \Phi_z\right) = \pi(1,1,2)$$

3D lattice

breaks inversion symmetry

Dubček, Kennedy, Lu,
Ketterle, Soljačić, Buljan
Phys. Rev. Lett. 114, 225301 (2015)

Weyl points: synthetic magnetic monopoles in momentum space

Inversion symmetry broken

$$H(\boldsymbol{k}) = -2\left(J_y \cos\left(k_y a\right) \sigma_x + K_x \sin(k_x a)\sigma_y + K_z \cos(k_z a)\, \sigma_z\right)$$

Two bands which touch at four Weyl points

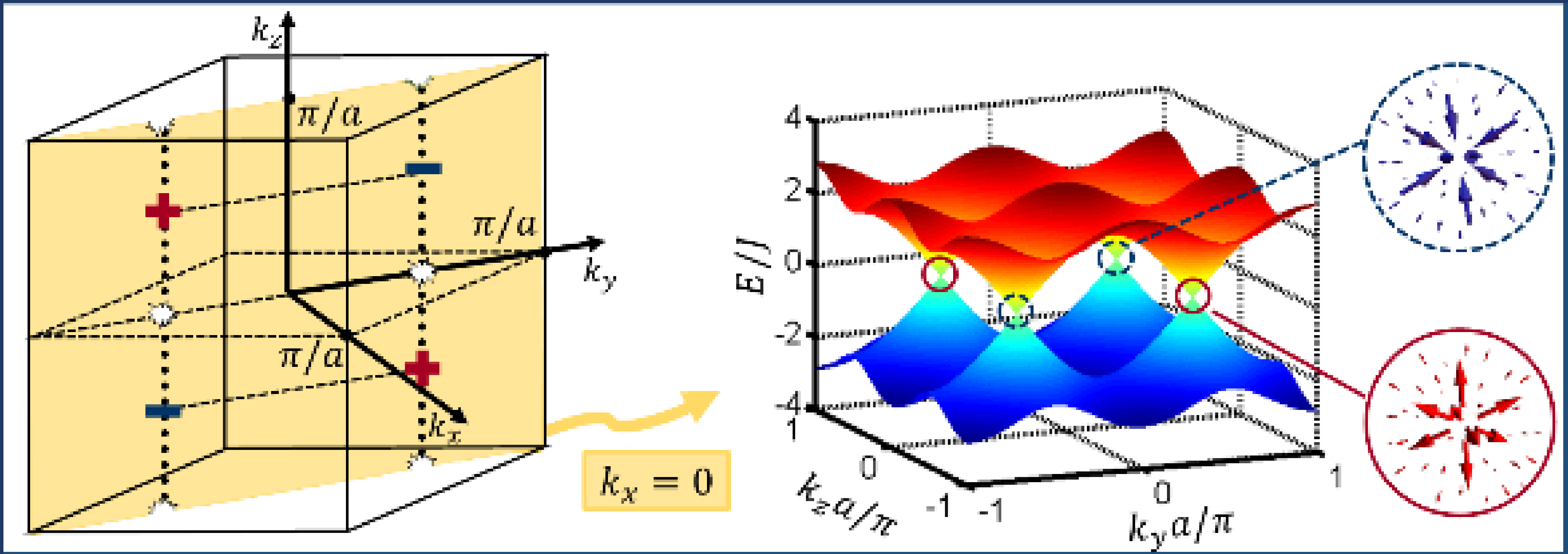
$$E(\boldsymbol{k}) = \pm 2\sqrt{K_x^2 \sin^2(k_x a) + J_y^2 \cos^2\left(k_y a\right) + K_z \cos^2(k_z a)}$$

Berry connection:

$$\mathbf{A}(\mathbf{k}) = i\langle u(\mathbf{k})|\nabla_{\mathbf{k}}|u(\mathbf{k})\rangle.$$

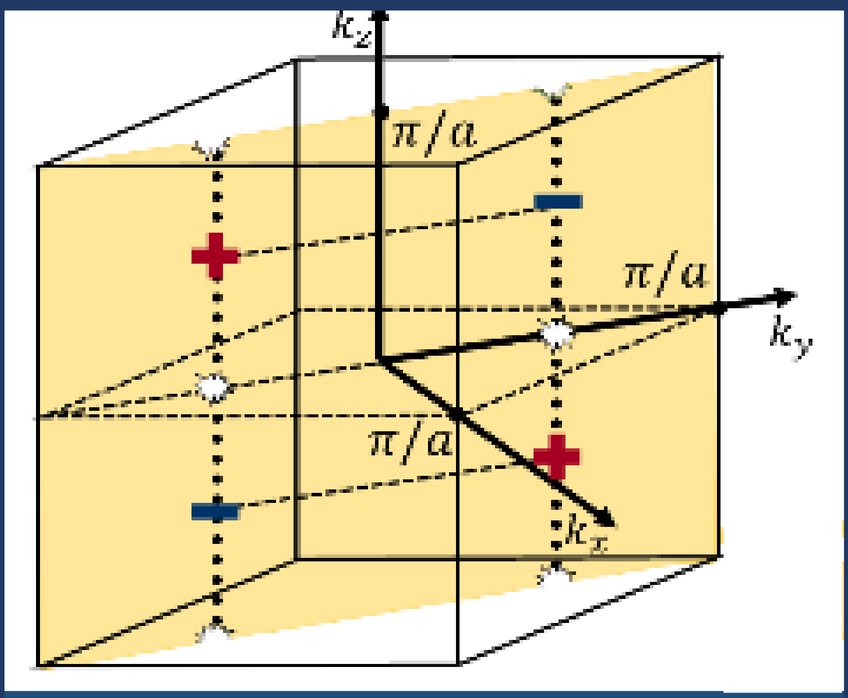
Berry curvature:

$$\mathbf{B} = \nabla_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})$$



Annihilation of Weyl points

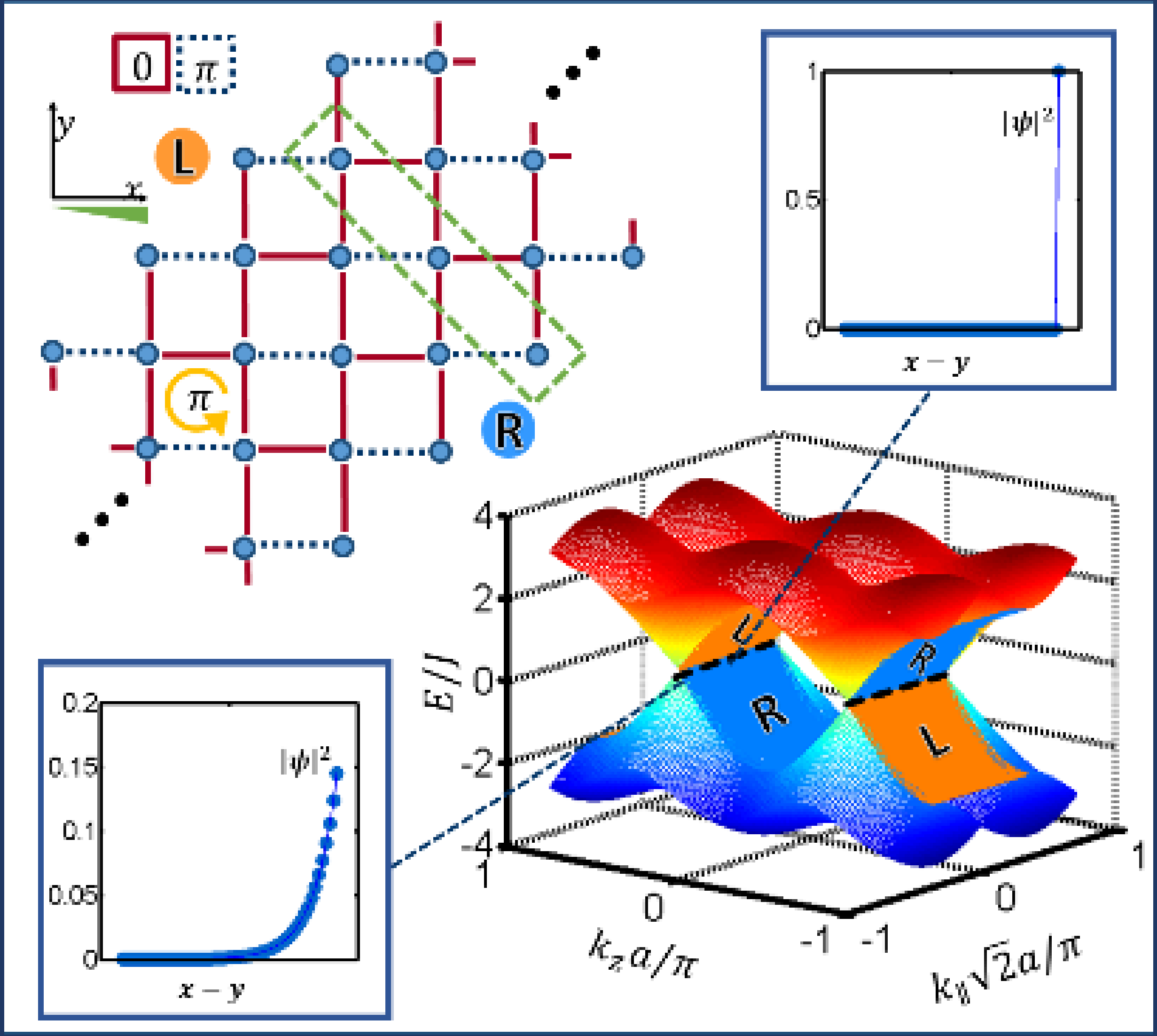
- Tunable A-B sublattice energy offset of on-site energies ($\pm\epsilon$)
- Additional $\epsilon\sigma_z$ term in Hamiltonian
- Weyl points with opposite chiralities annihilate for $\epsilon = \pm 2K_z$



Dubček, Kennedy, Lu,
Ketterle, Soljačić, Buljan
Phys. Rev. Lett. 114, 225301 (2015)

Fermi arc surface states

- Fermi arc surface states for a slab
- Dispersion sheets of surface states (on two sides of the slab) intersect along the Fermi arcs



The Quantum Hall Effect with Wilczek's charged magnetic flux tubes instead of electrons

Marija Todorić, Dario Jukić, Danko Radić, Marin Soljačić, Hrvoje Buljan,
In preparation

Anyons – fractional statistics

$$\psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = e^{i\pi\alpha}\psi(\boldsymbol{r}_2, \boldsymbol{r}_1)$$

FERMIONS $\alpha = 1$

BOSONS $\alpha = 0$

In two spatial dimensions, α can in principle take any value between 0 and 1

F. Wilczek, PRL 49, 957 (1982); J. Leinaas J. Myrheim, Nuovo Cimento B 37, 1 (1977)

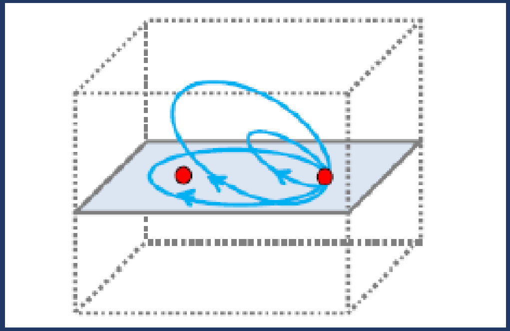
C. Nayak, S. H. Simon, A. Stern, M. Freedman, S. Das Sarma, RMP 80, (2008)

- potential platform – fault tolerant quantum computing

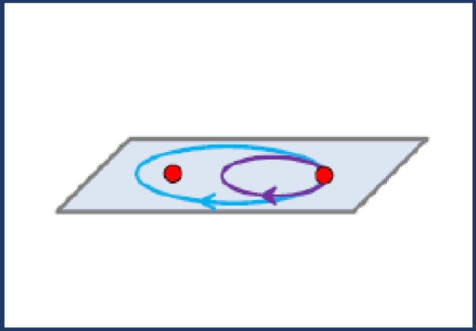
two exchanges = one particle encircles the other in the relative space

3D

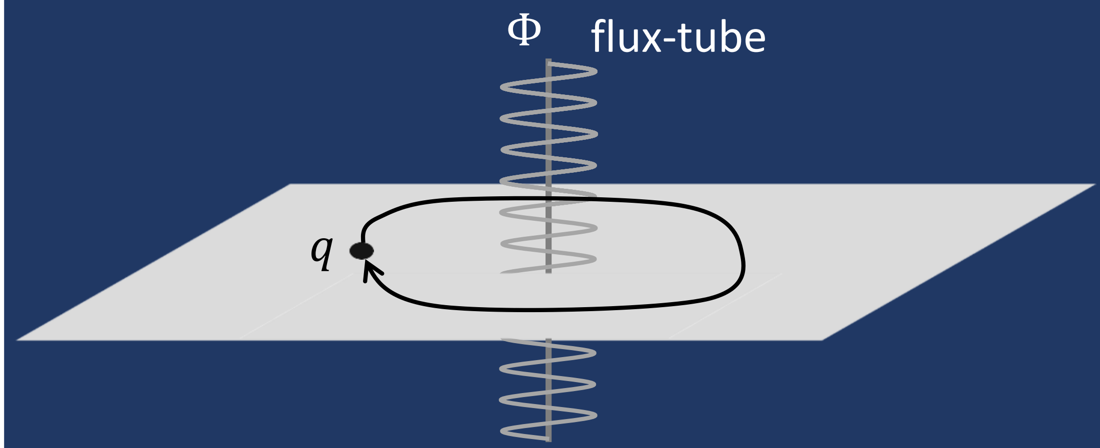
$$(e^{i\pi\alpha})^2 = 1$$
$$e^{i\pi\alpha} = \pm 1$$



2D



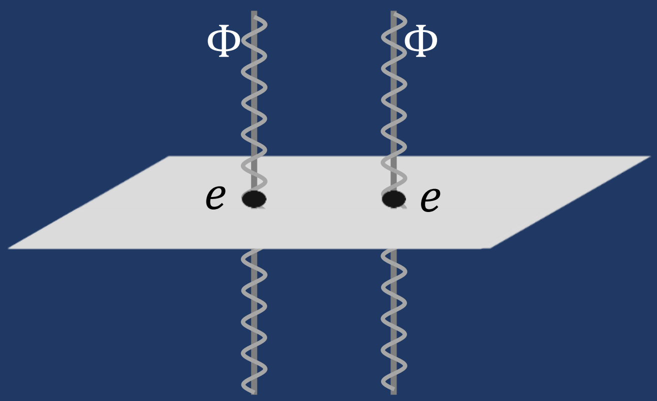
Wilczek's charged-flux-tube composites



Aharonov-Bohm phase

$$\varphi = \frac{q}{\hbar}\oint_P \vec{A}\cdot d\vec{l} = \frac{q}{\hbar}\phi$$

Wilczek's charged-flux-tube composites



Wilczek, PRL 49, 957 (1982)

$$H_{CP} = \sum_{i=1}^{n} \frac{1}{2m}\Big[\mathbf{p} - 2e\sum_{j\neq i}\mathbf{A}(\mathbf{r}_i-\mathbf{r}_j)\Big]^2$$

$$\mathbf{A}(\mathbf{r}_i-\mathbf{r}_j) = \frac{\Phi}{2\pi}\,\frac{\hat{z}\times(\mathbf{r}_i-\mathbf{r}_j)}{|\mathbf{r}_i-\mathbf{r}_j|^2}$$

$\psi_{CP}(\mathbf{r}_1,\ldots,\mathbf{r}_N,t)$ is bosonic or fermionic

Singular gauge transformation:

$$\psi(\mathbf{r}_1,\ldots,\mathbf{r}_N,t) = \prod_{i<j}^{N} e^{-i\phi_{ij}\Delta}\,\psi_{CP}$$

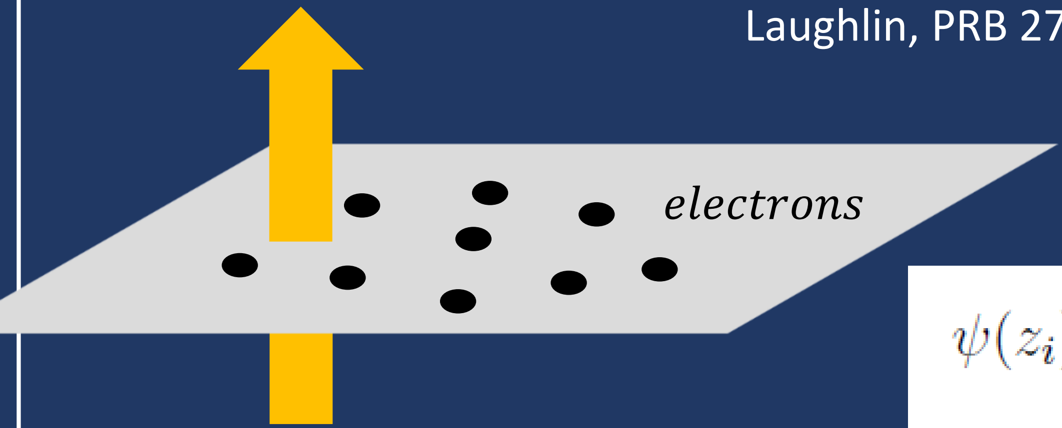
fractional statistics

$$\Delta = \frac{e}{\pi\hbar}\oint \mathbf{A}\cdot d\mathbf{l}$$

$$H = \sum_{i=1}^{n} \frac{1}{2m}\Big[\mathbf{p} - 2e\,\cancel{\sum_{j\neq i}\mathbf{A}(\mathbf{r}_i-\mathbf{r}_j)}\Big]^2$$

Anyons in the Fractional Quantum Hall Effect

Magnetic field, B

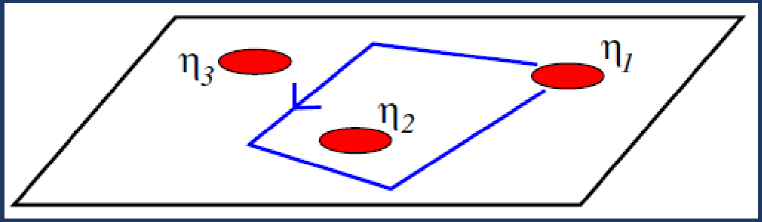


Tsui, Stormer, Gossard, PRL 48, 1559 (1982).
Laughlin, PRB 27, 3383 (1983); PRL 50, 1395 (1983).

- electron-electron interactions!
- Laughlin '83, $m = 1/\upsilon$

$$\psi(z_i) = \prod_{i<j} (z_i - z_j)^m e^{-\sum_{i=1}^{n} |z_i|^2/4l_B^2}$$

Quasiholes (excitations)

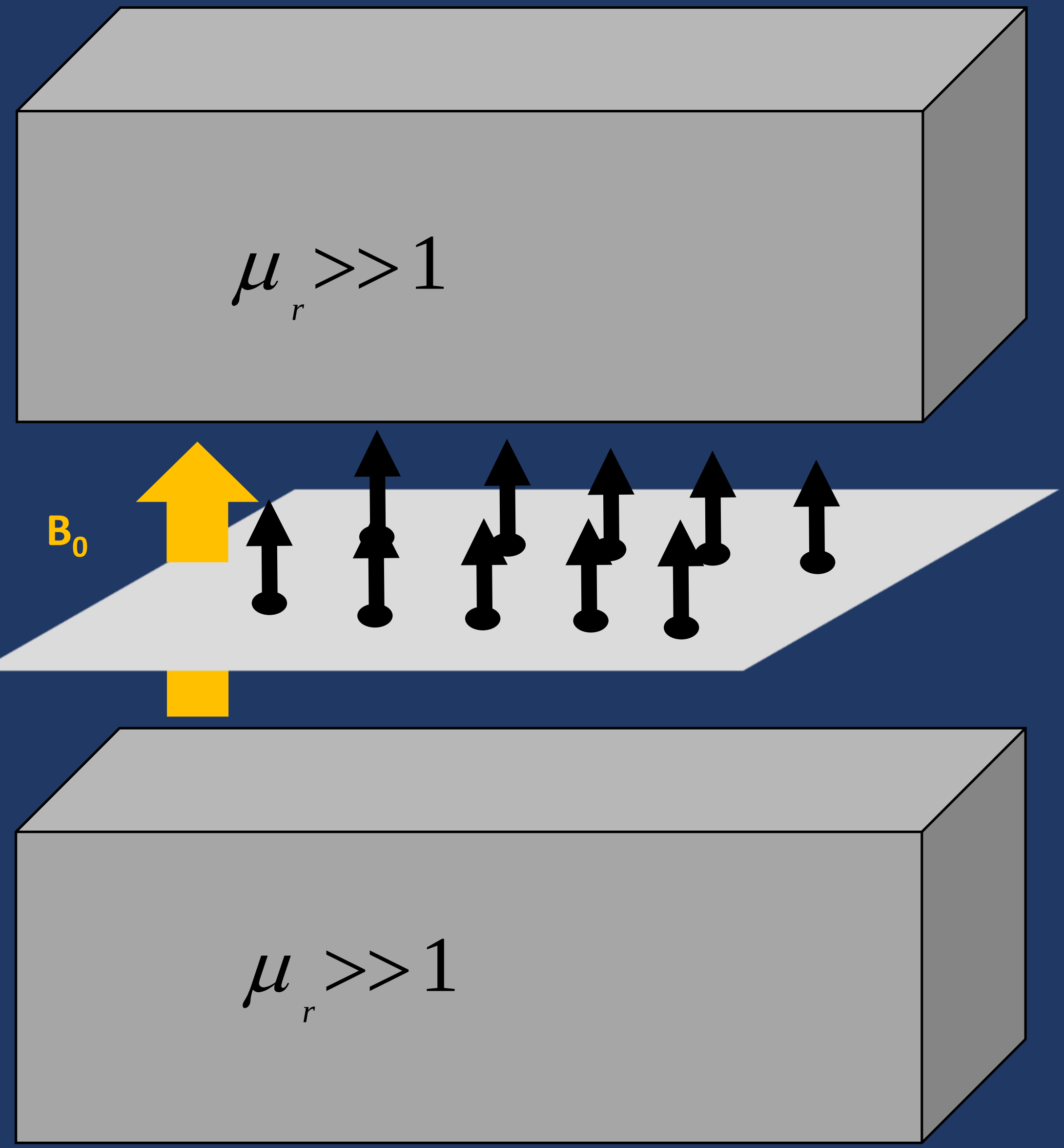


- **Fractional charge $e* = e/m$**
- **Fractional statistics – anyons**

Arovas, Schrieffer, Wilczek PRL 53, 722 (1984), $\boldsymbol{exp(2\pi i\alpha), \alpha = 1/m}$

$$\psi_{\mathrm{M-hole}}(z;\eta) = \prod_{j=1}^{M} \prod_{i=1}^{N} (z_i - \eta_j) \prod_{k<l} (z_k - z_l)^m \, e^{-\sum_{i=1}^{n} |z_i|^2/4l_B^2}$$

Proposals for realization of Wilczek's composites



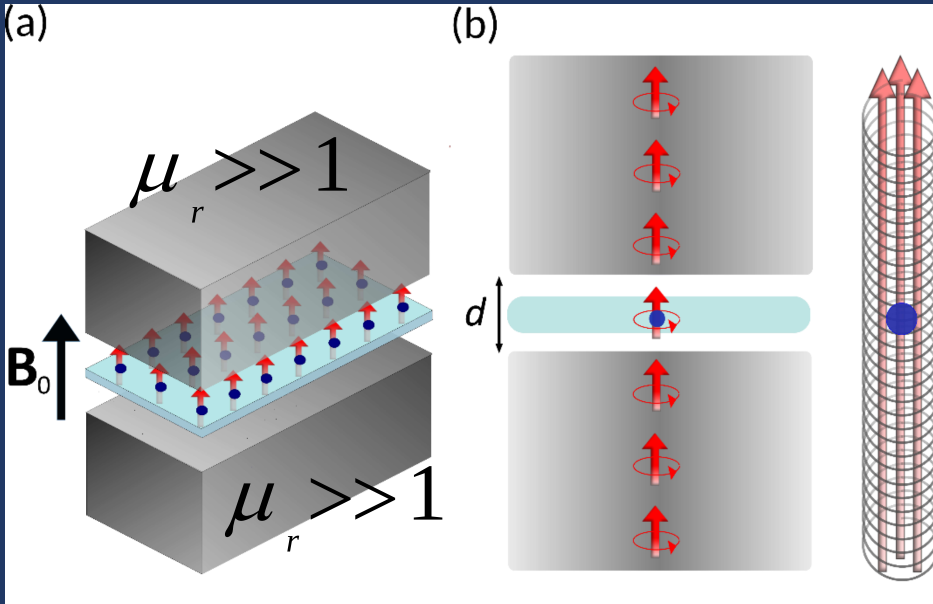
Starting assumptions:

- electrons in the INTEGER QHE state (say lowest Landau Level filled);Coulomb interactions neglected
- Magnetic moments of the electrons (arising from spin) aligned with the magnetic fields

System:

- Sandwich the 2D electron system between 2 blocks of high magnetic permeability metamaterials, w/ fast temporal response

e-e vector potential interactions

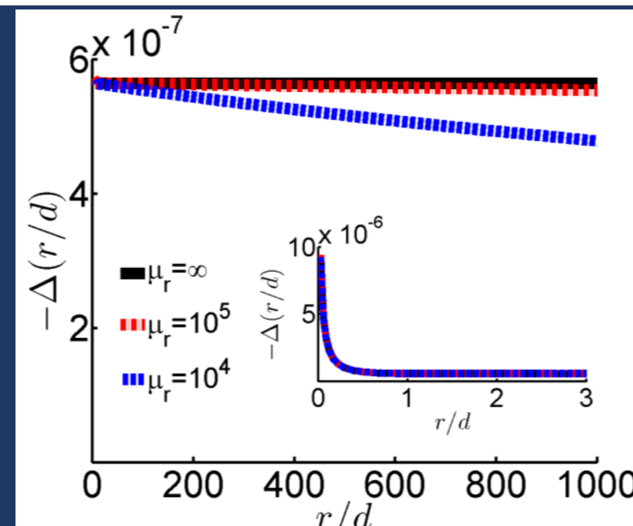


- $\mu_r = \infty$
- $\mathbf{A}(\mathbf{r}) \approx \frac{\Phi}{2\pi r}\hat{\phi}$
- effective flux tube

$$\Delta = \frac{q}{\pi\hbar}\oint \mathbf{A}\cdot d\mathbf{l}$$

$$\mathbf{A}(\mathbf{r}) \approx \Phi/2\pi r\hat{\phi}$$

$d = 10$ nm



Many-body Hamiltonian

$$H_{CP} = \sum_{i=1}^{n} \frac{1}{2m} \left[\mathbf{p}_i - q\mathbf{A}_0(\mathbf{r}_i) - 2q \sum_{j \neq i} \mathbf{A}(\mathbf{r}_i - \mathbf{r}_j)\right]^2$$

- $\mathbf{A}_0(\mathbf{r}) = \frac{1}{2}\, \mathbf{B}_0 \times \mathbf{r}$ $\quad \mathbf{A}(\mathbf{r}_i - \mathbf{r}_j) = \frac{\Phi}{2\pi} \frac{\hat{z} \times (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^2}$

mediated by the metamaterial

Related to composite fermions, Jain PRL 63, 199 (1989)

- singular gauge transformation

$$H = \sum_{i=1}^{n} \frac{1}{2m} \left[\mathbf{p}_i - q\mathbf{A}_0(\mathbf{r}_i)\right]^2$$

$$\psi(\mathbf{r}_1, \ldots, \mathbf{r}_N, t) = \prod_{i<j}^{N} e^{-i\phi_{ij}\Delta}\, \psi_{CP}$$

- φ_{ij} azimuthal angle of $\mathbf{r}_1 - \mathbf{r}_2$
- multivalued wave function

Signature of anyons

Slight shift of the plateau of the Integer Quantum Hall Effect

$$\psi(\{z_i\}\{z_i^*\}) = \prod_{i<j}(z_i - z_j)^{\alpha}\exp(-\frac{1}{4l_B^2}\sum_l |z_l|^2)$$

$$\boxed{\sigma_H = \frac{e^2}{\alpha h}}$$

$d = 10$ nm $\qquad \frac{1}{\alpha} = \frac{1}{1-\Delta} \approx 1 + \Delta \qquad \Delta \sim 10^{-7}$

http://physics.nist.gov/cgi-bin/cuu/Value?rk

Fundamental Physical Constants

von Klitzing constant R_K	
Value	25 812.807 4555 Ω
Standard uncertainty	0.000 0059 Ω
Relative standard uncertainty	2.3 x 10^{-10}
Concise form	25 812.807 4555(59) Ω

Discussion

- Characteristic time-scale in the QHE – cyclotron, Larmor frequencies

$$\omega_{cyclotron} = \frac{eB}{m^*} \sim \text{THz range}$$

- Material with $\mu_r(\omega \sim THz) \gg 1$

  Conventional materials tail of in GHz, hence metamaterials

  Pendry et al., IEEE Trans. Microw. Theory Tech. 47, 2075 (1999).
  Liberal et al. (Engheta group), Science 355, 1058 (2017)

- Choice of *d?*

$$\mathbf{A}(\mathbf{r}) \approx \Phi/2\pi r\hat{\phi}$$

  - good for $r > d$
  - d < average separation between electrons, for e density $10^{11} - 10^{12}\ \mathrm{cm}^{-2}$ it is 20 nm

- Heavy Fermion materials (to reduce $\omega_{cyclotron}$)?

Quasimomentum distribution and expansion of an anyonic gas

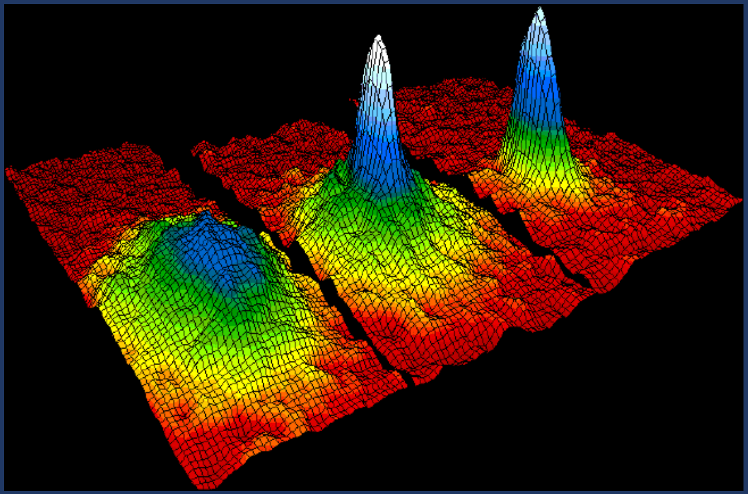
Tena Dubček, Bruno Klajn, Robert Pezer, Hrvoje Buljan, Dario Jukić,
arXiv:1707.04712 [cond-mat.quant-gas]

Momentum distribution in quantum many-body systems

- One of the key observables for describing a quantum many-body system

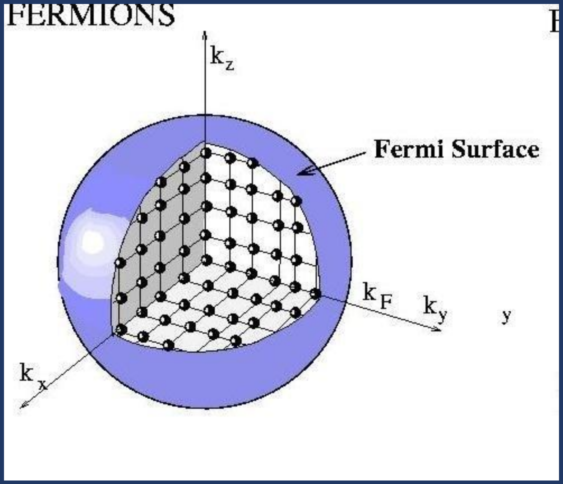
BOSONS

- **Example:** The experimental signature of Bose-Einstein condensation (BEC)



FERMIONS

- **Example:** Fermi surface in condensed matter physics



What about momentum distribution for ANYONS?

From: Many-body wavefunction (bosons & fermions)
To: Momentum distribution

1. Calculate the reduced-body density matrix (RSPDM)

$$\rho(\mathbf{r},\mathbf{r}',t)=N\int\psi^*(\mathbf{r},\mathbf{r}_2,\dots,\mathbf{r}_N,t)\psi(\mathbf{r}',\mathbf{r}_2,\dots,\mathbf{r}_N,t)d\mathbf{r}_2\dots d\mathbf{r}_N$$

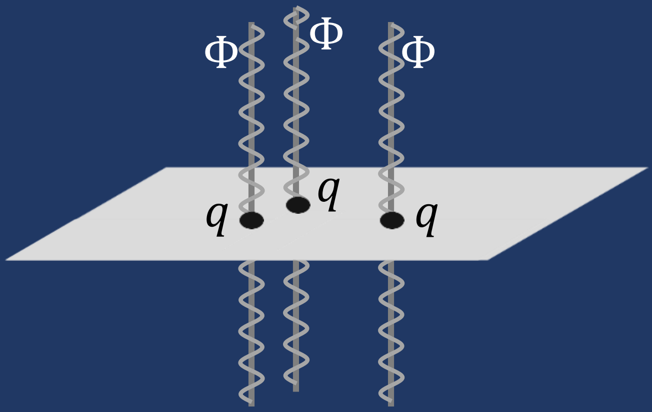
2. Calculate its Fourier transform (momentum representation)

$$n(\mathbf{k},t)=(2\pi)^{-2}\int\rho(\mathbf{r},\mathbf{r}',t)e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}d\mathbf{r}d\mathbf{r}'$$

It does not work for anyons!

- Anyonic wavefunction is multi-valued
- $n(\mathbf{k},t)$ WOULD NOT BE SINGLE VALUED !!!
- Not a proper observable !!!
- RSPDM: single-valued diagonal $\rho(\mathbf{r},t)\equiv\rho(\mathbf{r},\mathbf{r},t)$ - x-space single-particle density

Wilczek's composite particles – charged flux-tubes

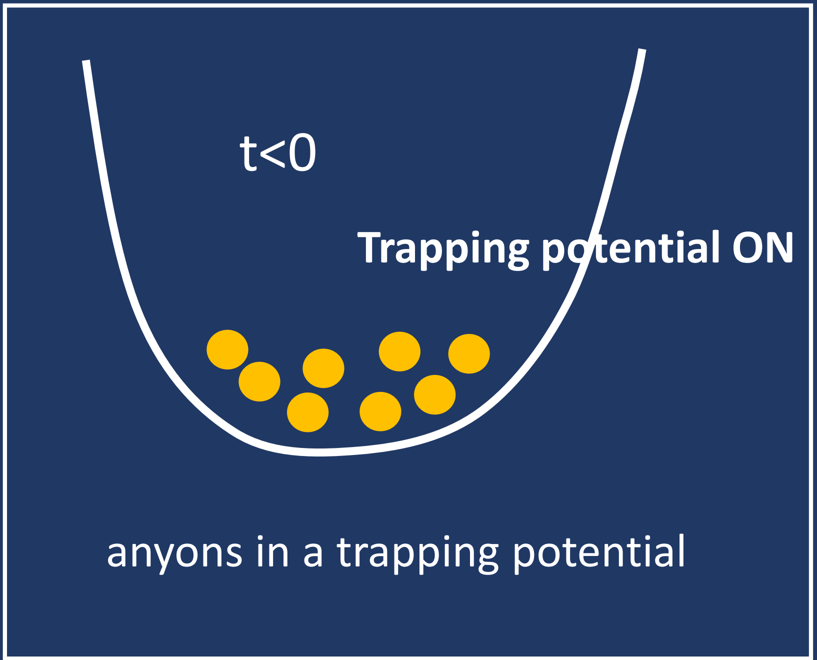


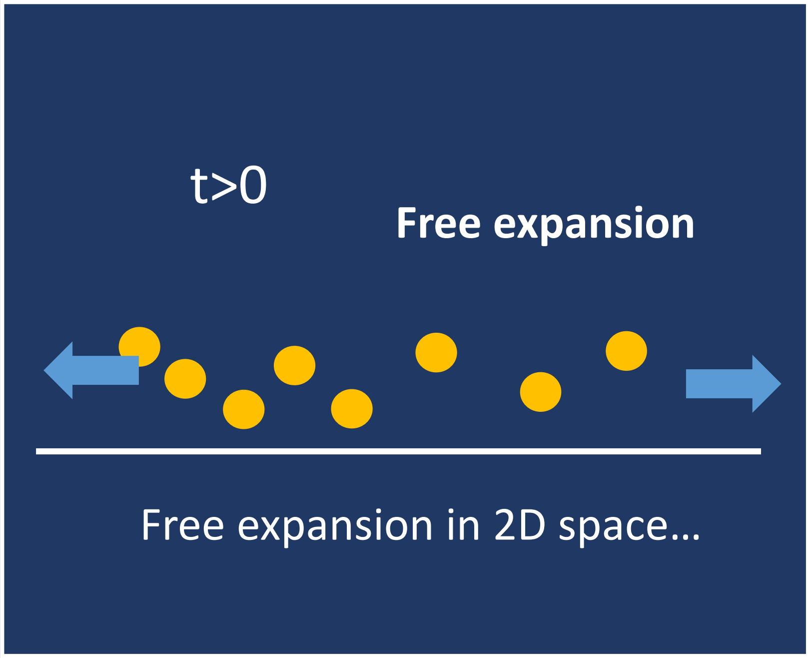
vector potential interactions

$$H_{CP} = \sum_{i=1}^{N} \left[-\frac{1}{2} \left(\nabla_i + i\alpha \sum_{j \neq i} \frac{\hat{\mathbf{z}} \times \mathbf{r}_{ij}}{r_{ij}^2} \right)^2 + \frac{1}{2} \omega^2(t) r_i^2 \right]$$

- Wavefunction $\psi_{CP}(\mathbf{r}_1, \ldots, \mathbf{r}_N, t)$ is single valued (bosonic or fermionic)
- RSPDM $\rho(\mathbf{r}, \mathbf{r}', t)$ single-valued, $n(\mathbf{k}, t)$ single valued
- But, $n(\mathbf{k}, t)$ CANONICAL and NOT KINETIC MOMENTUM DISTRIBUTION (depends on gauge)
- One cannot gauge out vector potential at the positions of the flux tubes!!!

What about free expansion (time-of-flight)?





CAN WE IDENTIFY ASYMPTOTIC SINGLE-PARTICLE DENSITY WITH QUASIMOMENTUM DISTRIBUTION?

(for FERMIONS and BOSONS this is the case)

Expansion of two anyons (N=2)

- definition reduces to the standard one when the statistical parameter approaches 0 for bosons or 1 for fermions
- asymptotic form of the single-particle density $\rho(r, t \to \infty)$ has the same shape as $|a_{Kk}|^2$
- quasimomentum distribution does not change during free expansion

Projection coefficients	$a_{Kk} \propto k^{|\alpha|} e^{-\frac{K^2}{4} - k^2}$

We identify $|a_{Kk}|^2$ with the quasimomentum distribution for 2 anyons

Expansion of N anyons

Initial state (t=0) is eigenstate in H.O.

$$\psi(\{\mathbf{r}_i\}, t{=}0) = \mathcal{N}_N \prod_{i<j} r_{ij}^{|\alpha|} e^{i\alpha\phi_{ij}} e^{-\sum_{k=1}^{N} \frac{|\mathbf{r}_k|^2}{2}}$$

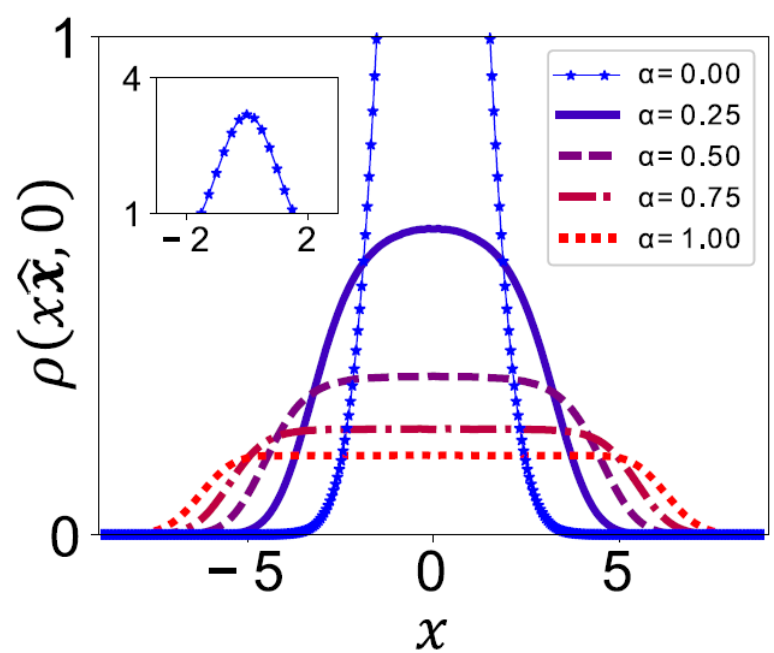
Time evolving state (t>0); found by scaling transf.

$$\psi(\{\mathbf{r}_i\}, t{>}0) = \frac{1}{b^N}\psi(\{\frac{\mathbf{r}_i}{b}\}, 0) e^{i\frac{\dot{b}}{2b}\sum_k^N |\mathbf{r}_k|^2} e^{-iE_N\tau(t)}$$

QUASIMOMENTUM DISTRIBUTION

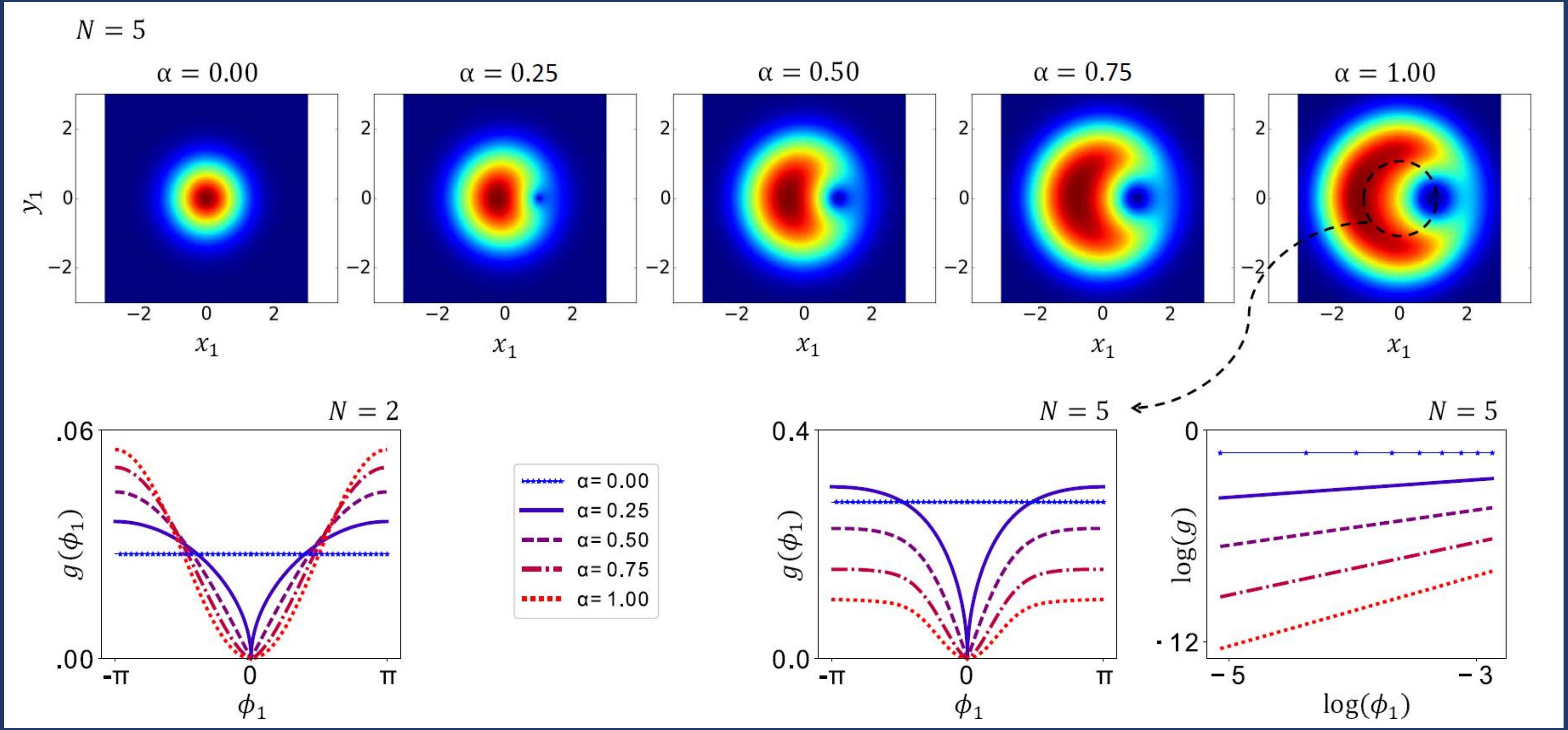
Asymptotic form of the single-particle density

$\rho(r, t \rightarrow \infty)$



Pair-correlation function

$$g(\mathbf{r}_1, \mathbf{r}_2, t) = N(N-1) \int |\psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N, t)|^2 d\mathbf{r}_3 \dots d\mathbf{r}_N$$

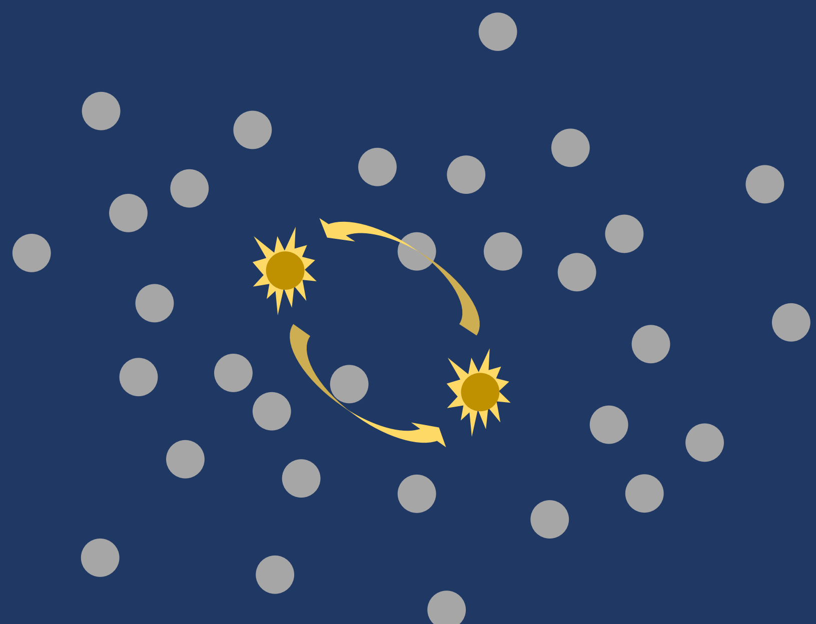


At small particle distances power-law scaling $g \sim |\boldsymbol{r}_1 - \boldsymbol{r}_2|^{2|\alpha|}$

Potential realisation

- hard core bosons in synthetic magnetic field

  in the FQHE state
- quasi-hole fractionalized excitations around new species of bosons
- repulsive hard-core interactions



Paredes, et al.,
Phys. Rev. Lett. 87, 010402 (2001).

Zhang, et al.,
Phys. Rev. Lett. 113, 160404 (2014).

Conclusion and Outlook

- **Weyl points – synthetic magnetic monopoles in k-space – accessible with ultracold atomic systems**
  Outlook Weyl: Include interactions in studies of Weyl points

- **Anyons:**
  (i) development of proposals for their observation
  - Wilczek's charged flux-tubes via IQHE & metamaterials

  (ii) theoretical understanding (observables, ground states)
  - Momentum distribution not a proper observable
  - Free expansion in 2D can provide insight

Photonics:

The Harper-Hofstadter Hamiltonian and conical diffraction in photonic lattices with grating assisted tunneling by T. Dubček, K. Lelas, D. Jukić, R. Pezer, M. Soljačić & H.B., New J. Phys. 17, 125002 (2015)

Four-dimensional photonic lattices and discrete tesseract solitons by D. Jukić and H. Buljan, Physical Review A 87, 013814 (2013) (synthetic dimension)