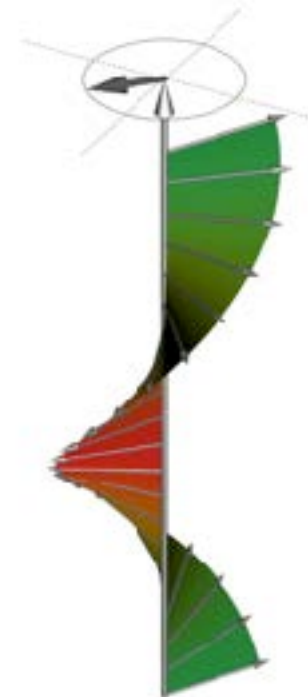
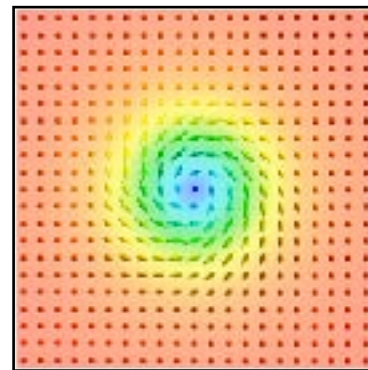
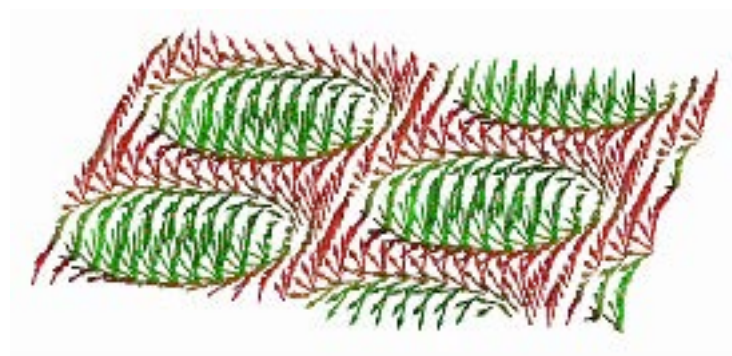


Magnonics in skyrmion-hosting chiral magnetic materials

Markus Garst

TU Dresden



Collaboration

theory:

Johannes Waizner



Achim Rosch (Köln)



experimental groups:

Peter Böni (München)

Neutron scattering



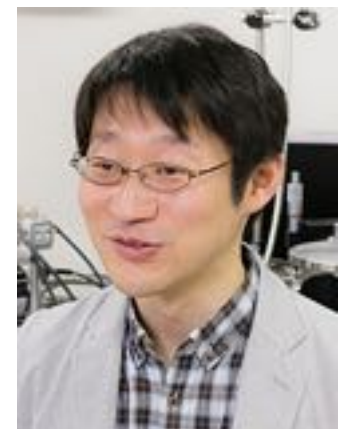
Dirk Grundler (Lausanne)

Magnetic resonance



Shinichiro Seki (Riken)

Spinwave spectroscopy



Outline:

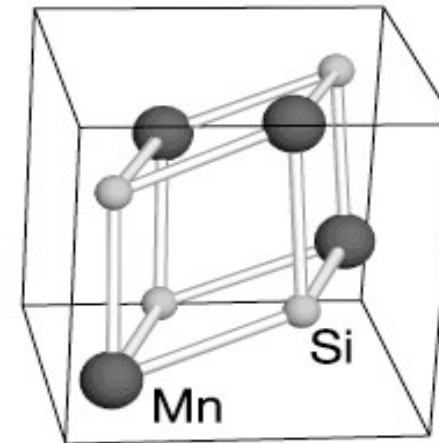
- Introduction to chiral magnets
- Spin-wave dynamics of the magnetic helix
- Spin-wave dynamics of the magnetic skyrmion
- Spin-wave dynamics of the magnetic skyrmion lattice

Introduction to chiral magnets

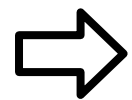
Chiral magnets

non-centrosymmetric, cubic magnets: MnSi, FeGe, $\text{Fe}_x\text{Co}_{1-x}\text{Si}$, Cu_2OSeO_3 , ...

Bravais lattice: simple cubic
space group: $P2_13$ (B20)



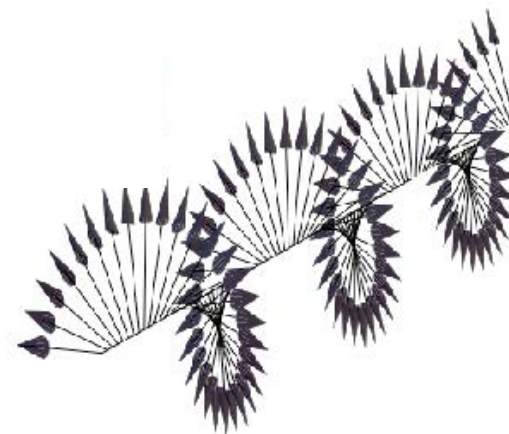
chiral atomic
crystal lattice



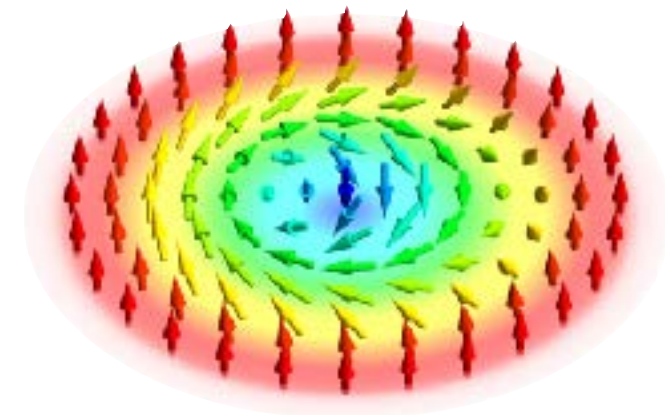
$D\vec{M}(\nabla \times \vec{M})$
Dzyaloshinskii-Moriya interaction

crystal chirality inherited by magnetism

chiral magnetic textures



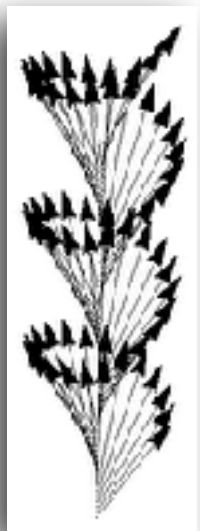
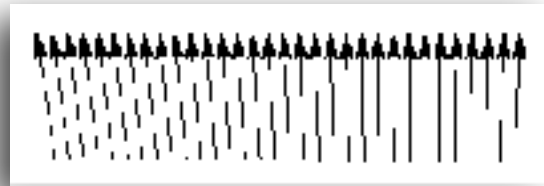
helices



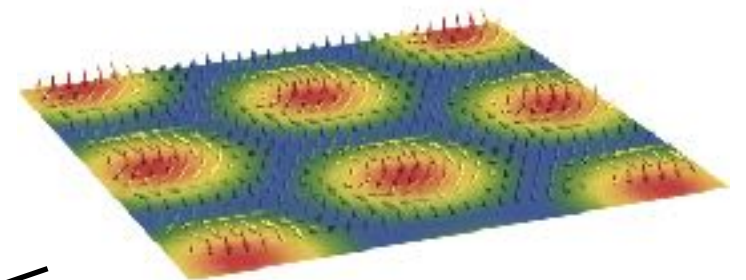
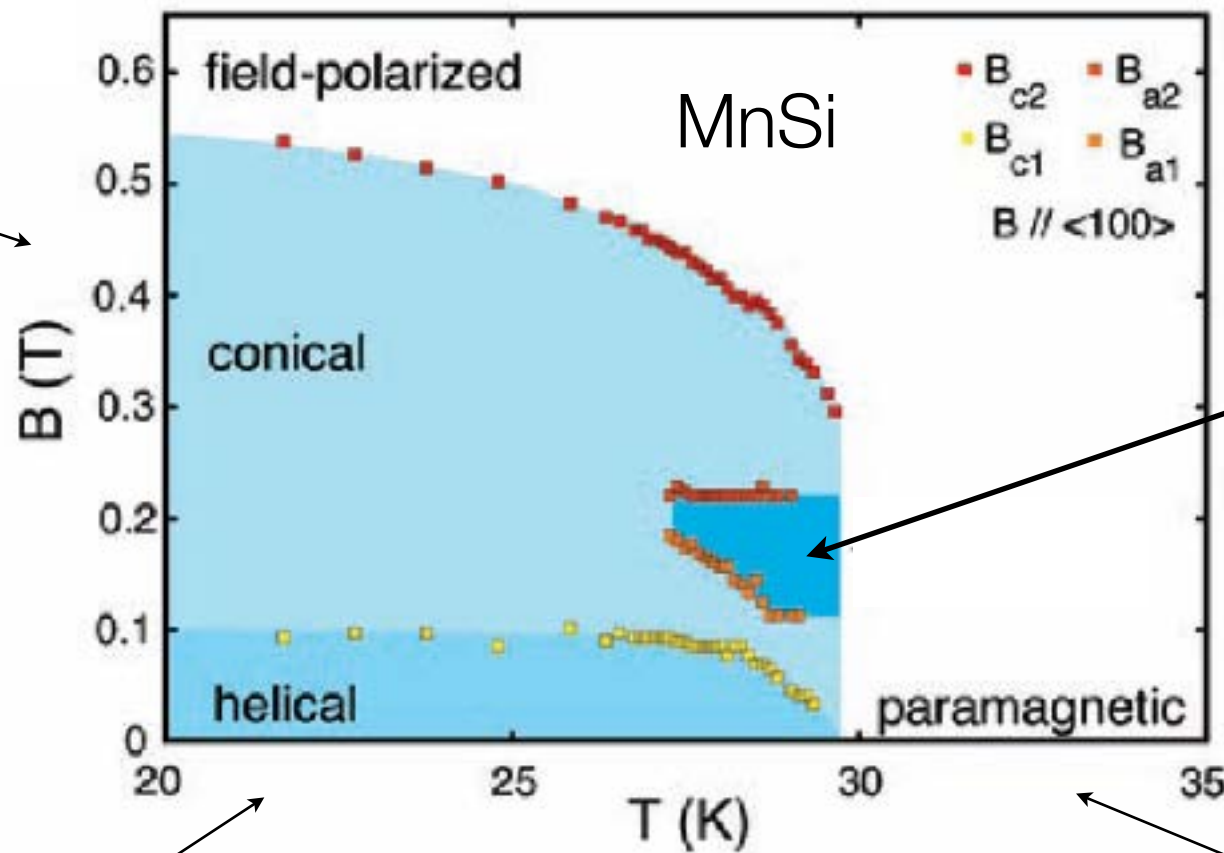
skyrmions

Phase diagram of chiral magnets

example: MnSi

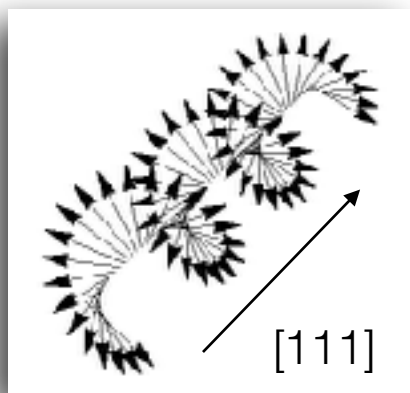


B

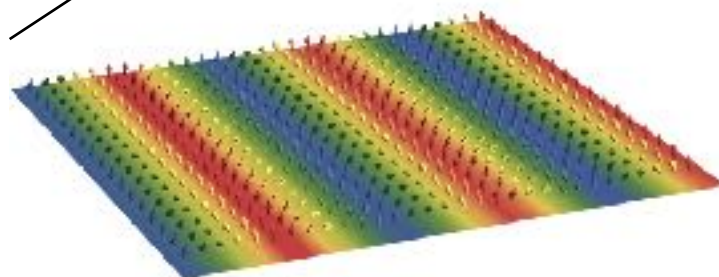


Skyrmion crystal

S. Mühlbauer et al. Science (2009)



helix



Skyrmions

Tony Skyrme (1961,1962)

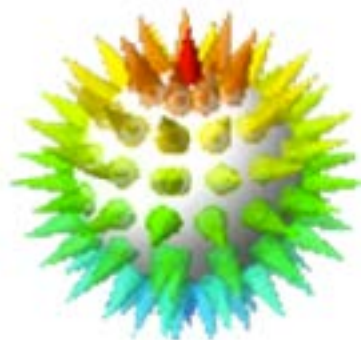
solutions of a non-linear field theory,
model for baryons



B=3

(isospin doublet ${}^3\text{H}/{}^3\text{He}$)

stereographic projection from sphere to plane: $\Pi_2(S^2) = \mathbb{Z}$



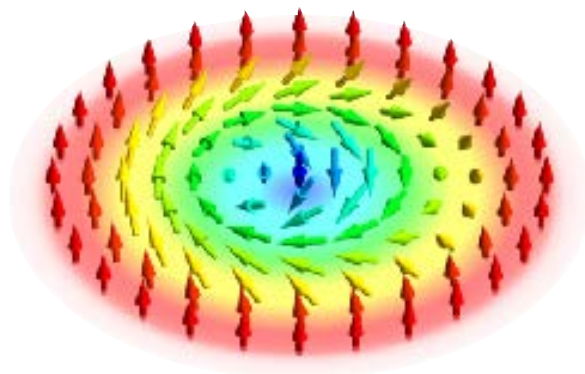
hedgehog



topologically stable object with
quantized **winding number**

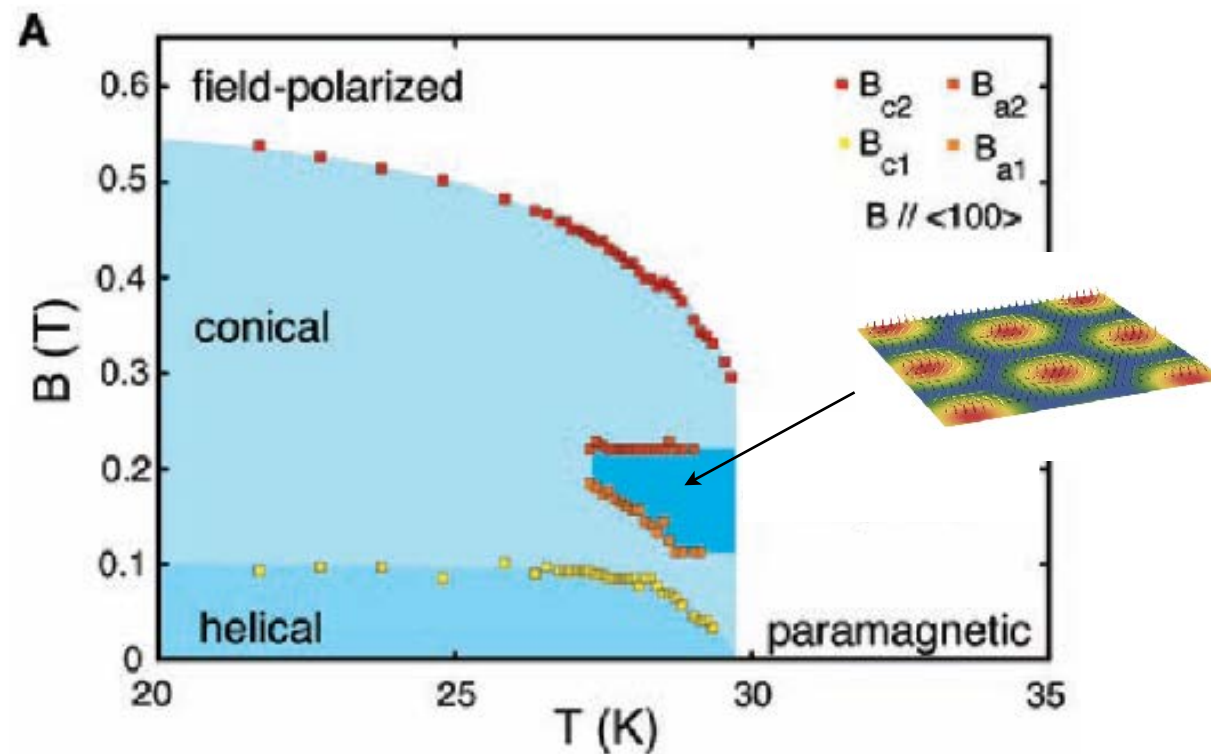
$$W = \frac{1}{4\pi} \int d^2\mathbf{r} \hat{M} \left(\partial_x \hat{M} \times \partial_y \hat{M} \right)$$

essential for skyrmion dynamics!

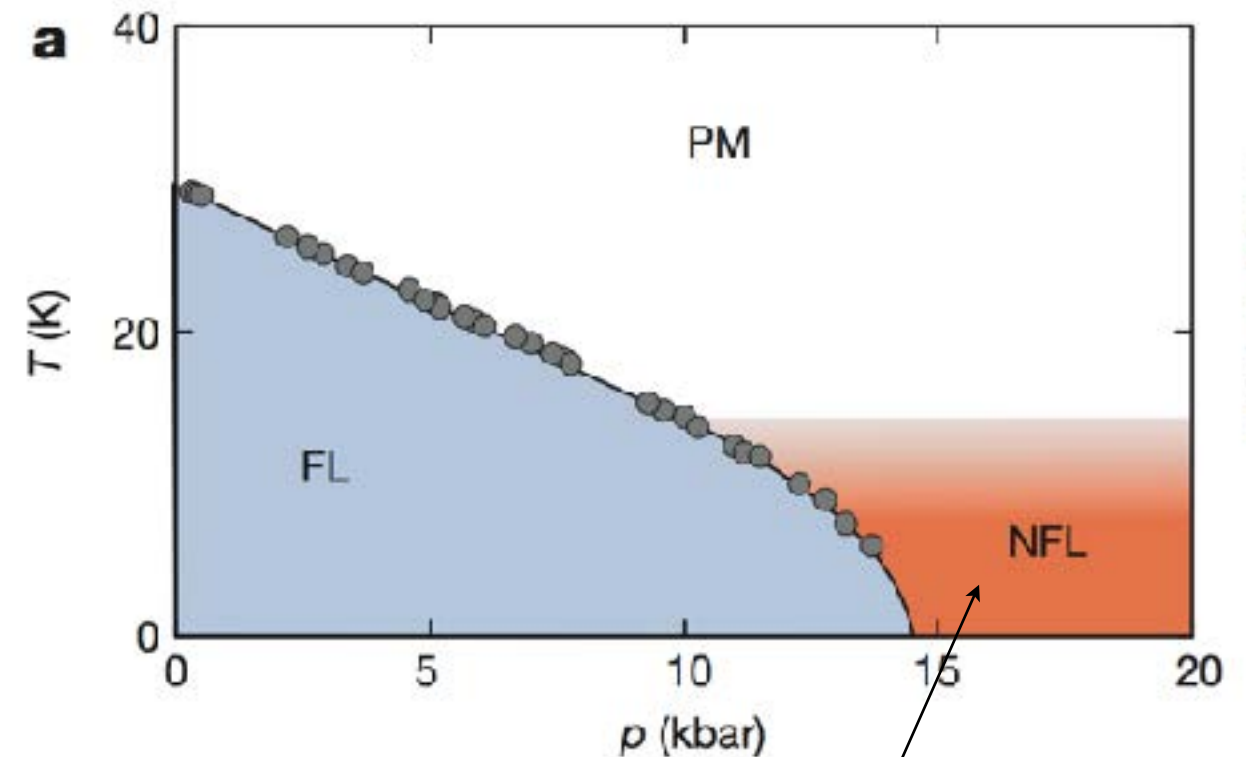


Phase diagram of MnSi con't.

in a magnetic field



under pressure



unexplained mystery!

slow magnetisation dynamics
destroying the Fermi liquid?

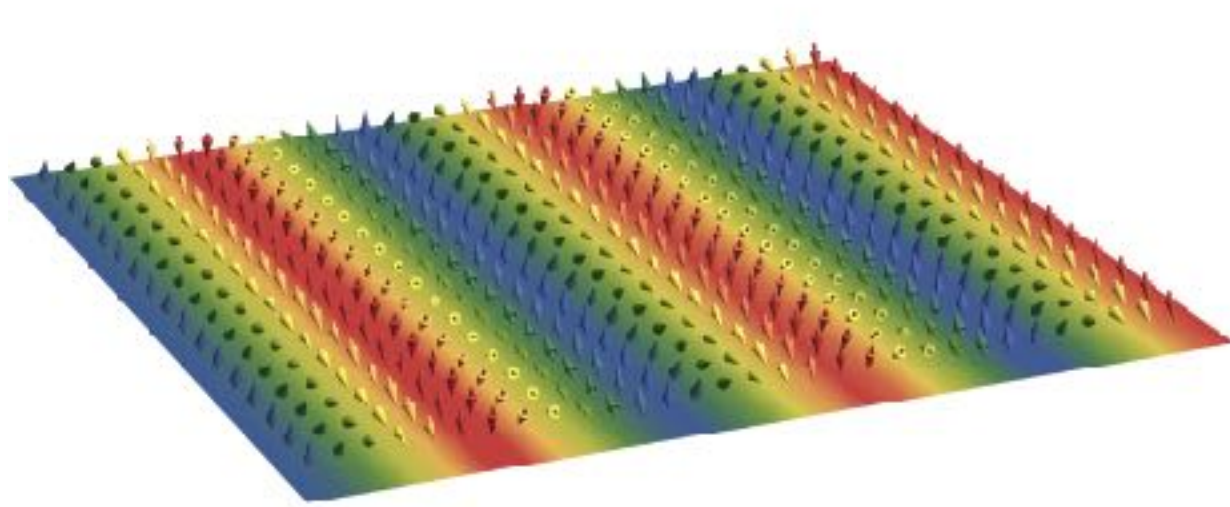
resistivity $\delta\rho \sim T^{3/2}$

extended non-Fermi liquid regime
three (!) decades in temperature

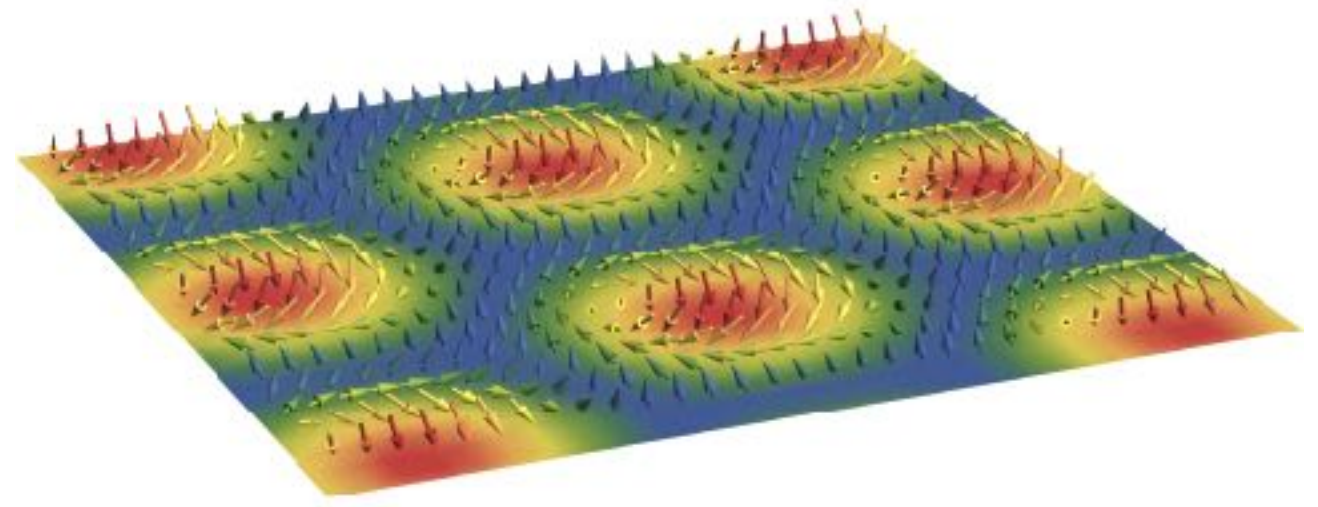
Pfleiderer, Julian, Lonzarich, Nature 2001
Ritz ... Pfleiderer, Nature 2013

This talk:

Spin-wave dynamics of chiral magnetic textures



1d helix



2d skyrmion lattice

wavelength: $\lambda \approx 18$ nm in MnSi
70 nm in FeGe

Magnetization dynamics

Landau-Lifshitz-Gilbert equation

$$\partial_t \vec{M} = -\gamma \vec{M} \times \vec{B}_{\text{eff}} + \dots$$

precession
in effective field

damping, driving
currents etc.

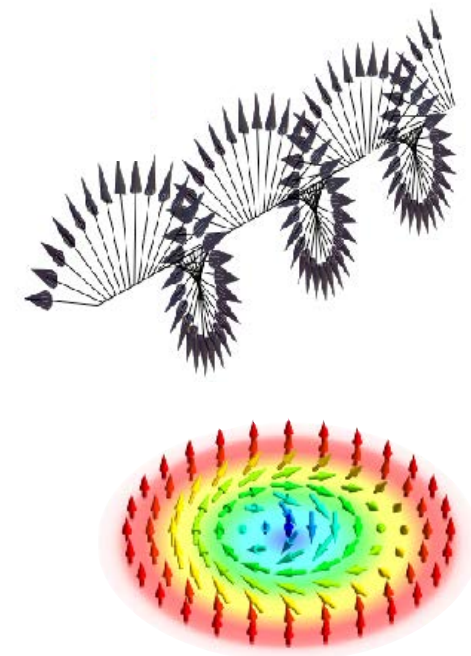


wikipedia

effective field is determined by the magnetic texture:

$$\vec{B}_{\text{eff}} = -\frac{1}{M_s} \frac{\delta F}{\delta \hat{M}}$$

with the energy functional $F = \int d\vec{r} \mathcal{V}(\hat{M})$



Linear spin-wave theory

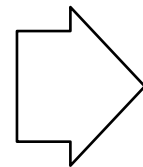
small amplitude excitation of the equilibrium magnetisation texture \hat{M}_{eq}

$$\hat{M} = \hat{M}_{\text{eq}} \sqrt{1 - 2|\psi|^2} + \hat{e}^+ \psi + \hat{e}^- \psi^*$$

with local orthogonal frame $\hat{e}_1(\mathbf{r}) \times \hat{e}_2(\mathbf{r}) = \hat{M}_{\text{eq}}(\mathbf{r})$
 $\hat{e}_{\pm} = \frac{1}{\sqrt{2}}(\hat{e}_1 \pm i\hat{e}_2)$

Landau-Lifshitz equation

$$\partial_t \vec{M} = -\gamma \vec{M} \times \vec{B}_{\text{eff}}$$



$$i\hbar\tau^z \partial_t \vec{\Psi} = H \vec{\Psi}$$

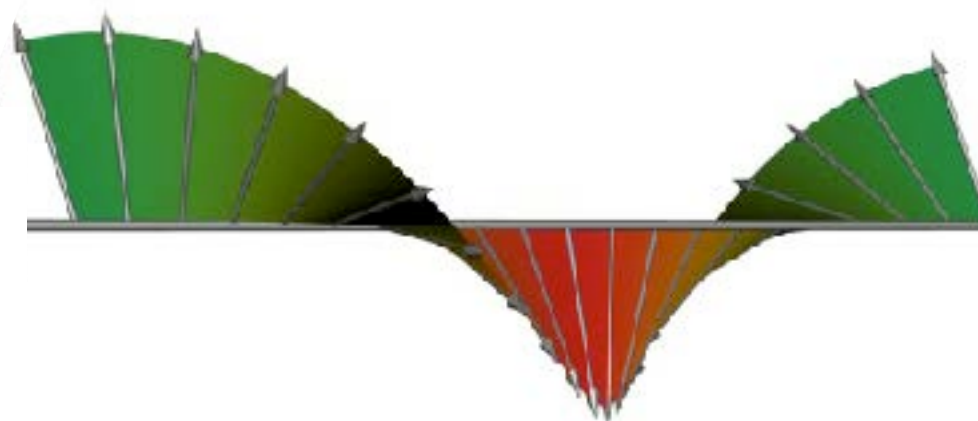
magnon spinor wave function $\vec{\Psi}^T = (\psi, \psi^*)$

U(1) charge = spin angular momentum of magnon not conserved!

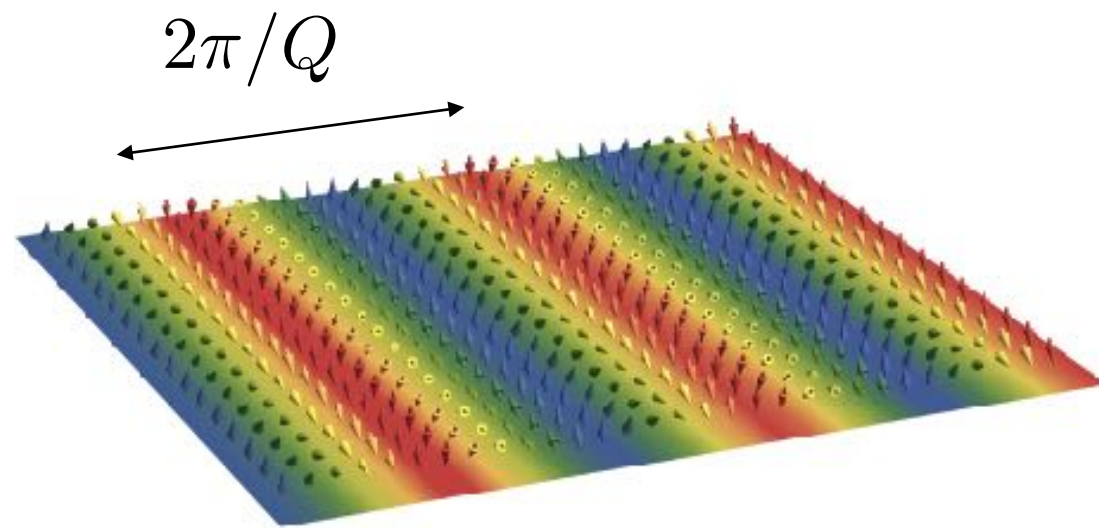
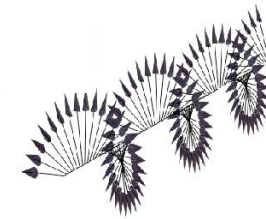
due to spin-orbit coupling, texture and dipolar interactions

Bogoliubov-deGennes 2x2 matrix Hamiltonian H

Spin-wave dynamics of the magnetic helix



Magnon excitations of the magnetic helix



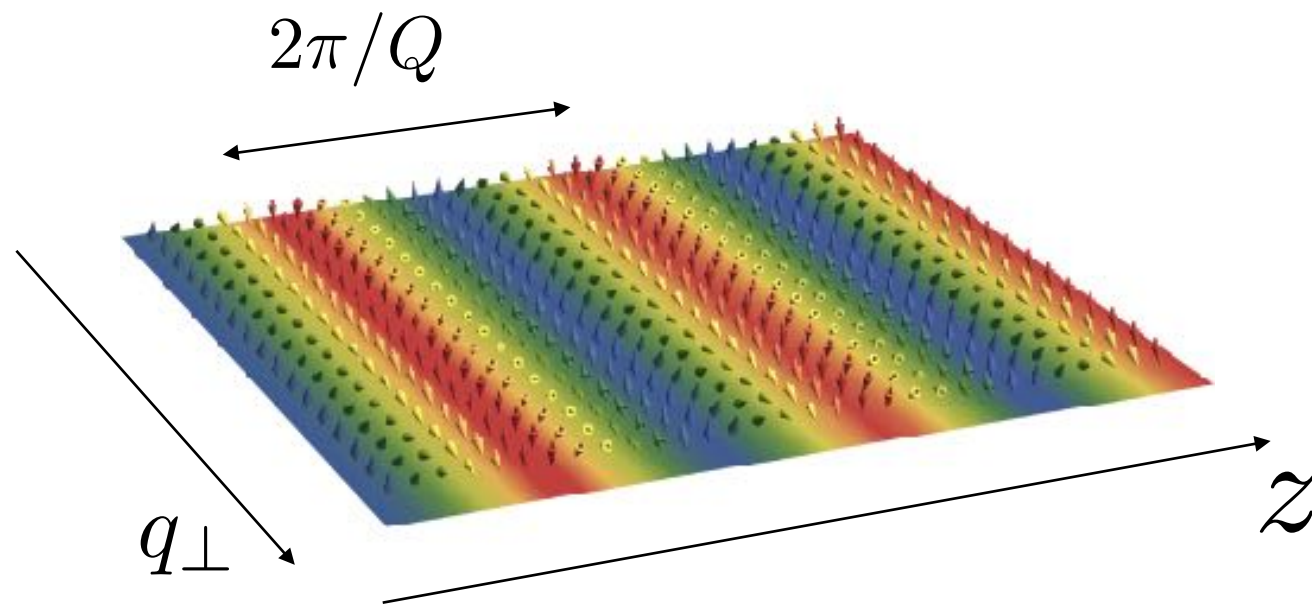
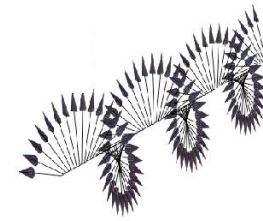
helix = 1d magnetic crystal

magnon excitations should obey Bloch's theorem

⇒ magnon band structure

Kugler, MG *et al* PRL (2015)

Magnon excitations of the magnetic helix



helix = 1d magnetic crystal

magnon excitations should obey Bloch's theorem

⇒ magnon band structure

magnon wave equation $i\hbar\tau^z\partial_t\vec{\Psi} = \mathcal{H}\vec{\Psi}$ for spinor $\vec{\Psi} = (\psi, \psi^*)$

with the magnon Hamiltonian:

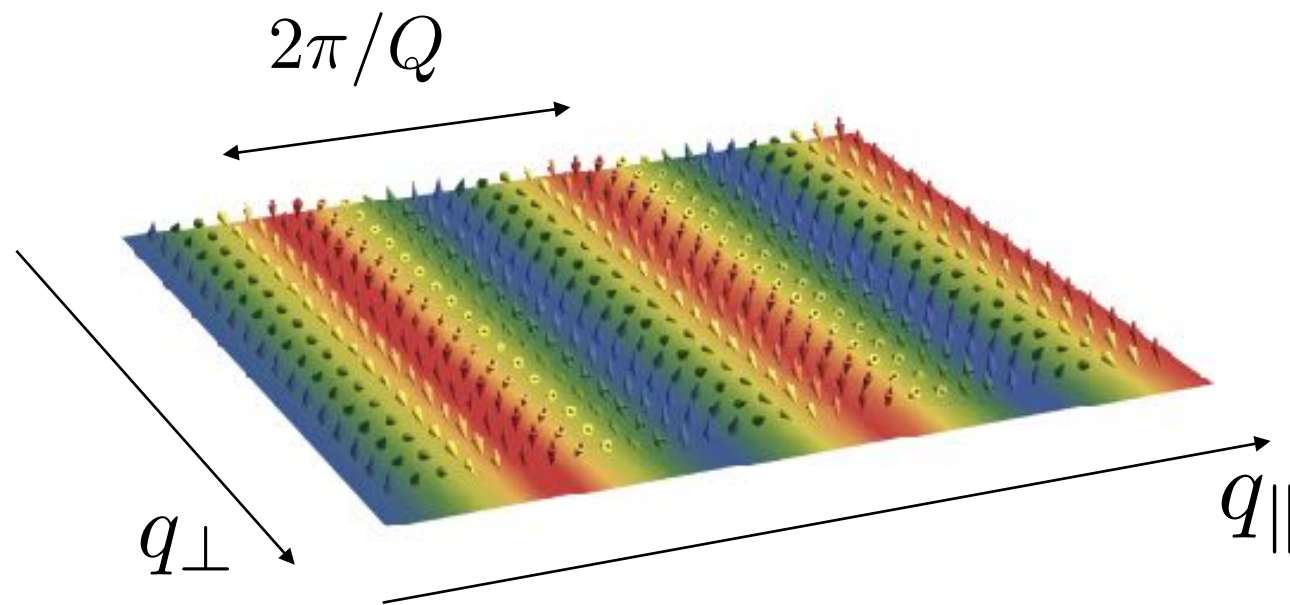
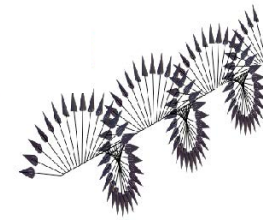
$$\mathcal{H}_0 = \mathcal{D} \left[\mathbb{1}(q_{\perp}^2 - \partial_z^2) - i2\tau^z Q q_{\perp} \cos(Qz) + \frac{Q^2}{2}(\mathbb{1} - \tau^x) \right]$$

variant of the Mathieu equation

particle in a one-dimensional periodic cosine potential

Kugler, MG *et al* PRL (2015)

Magnon excitations of the magnetic helix

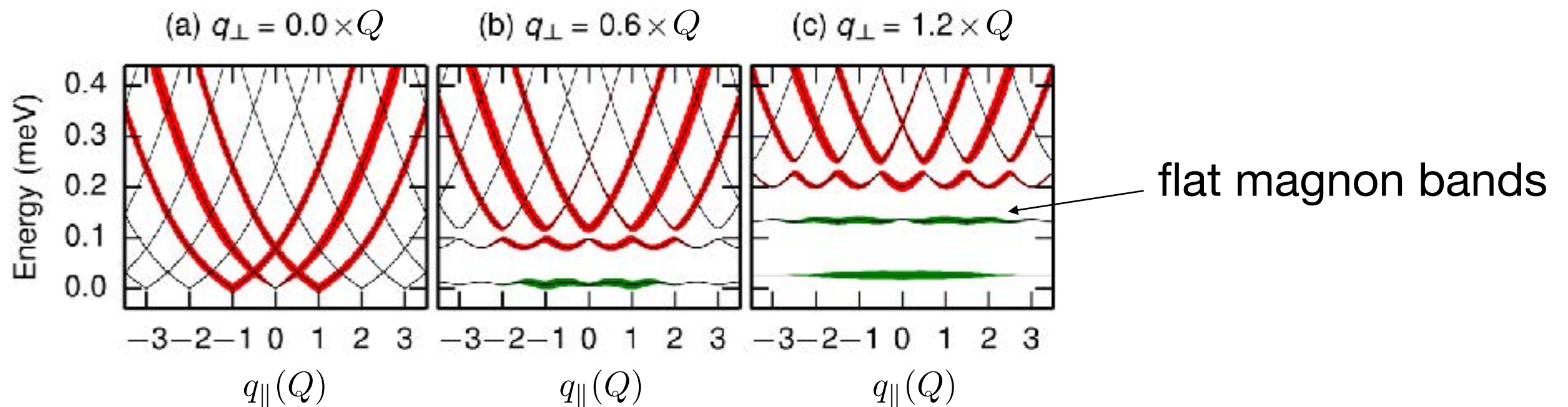


helix = 1d magnetic crystal

magnon excitations should obey Bloch's theorem

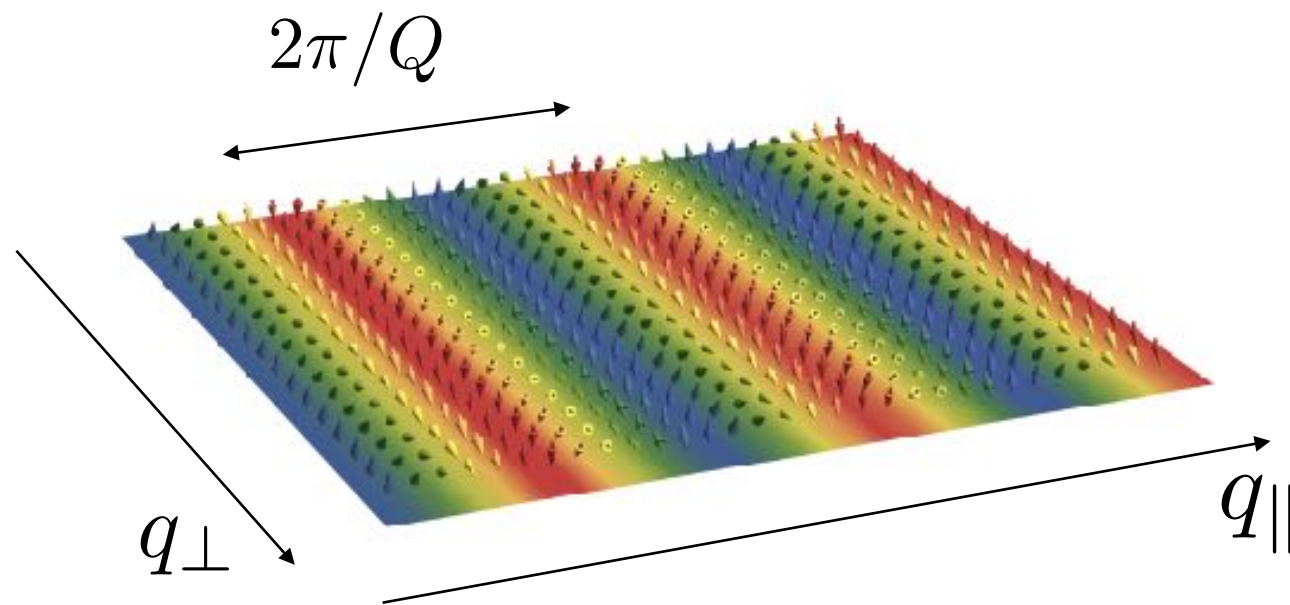
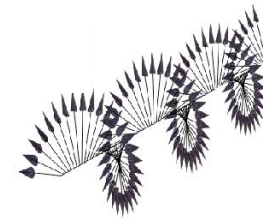
⇒ magnon band structure

transversal momentum q_{\perp} tunes strength of periodic potential
crossover from **weak** to **tight-binding** limit



Kugler, MG *et al* PRL (2015)

Magnon excitations of the magnetic helix

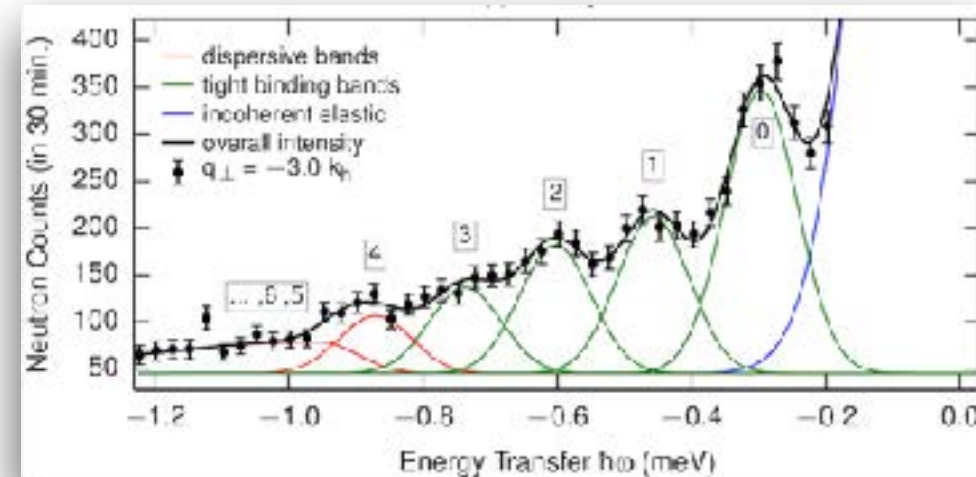
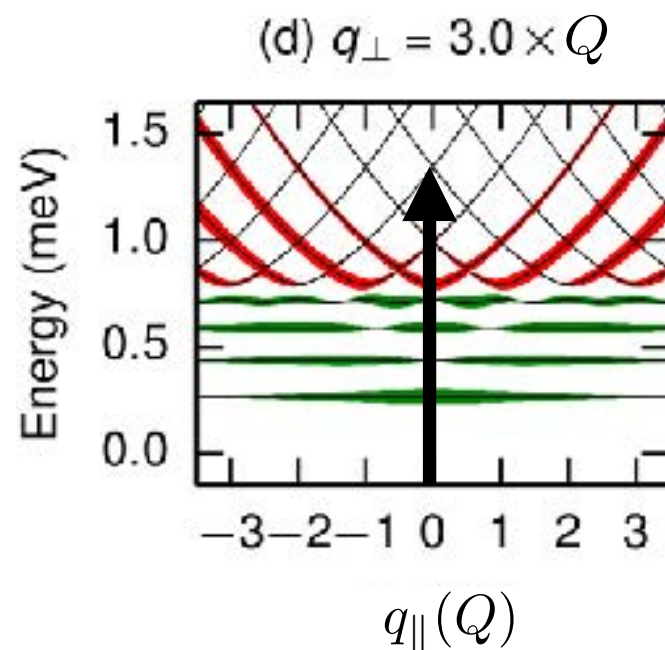


helix = 1d magnetic crystal

magnon excitations should obey Bloch's theorem

⇒ magnon band structure

Inelastic neutron scattering on MnSi:

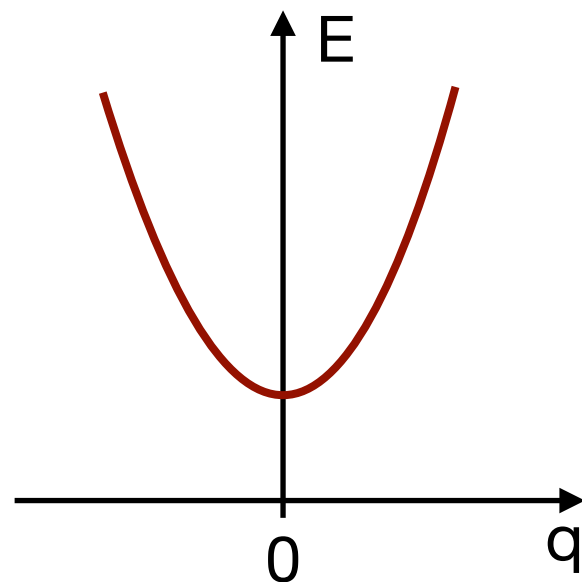
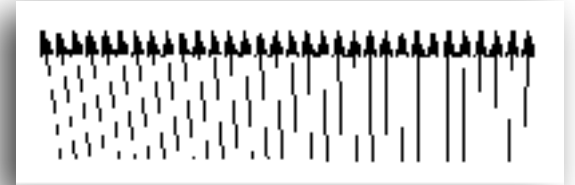


five magnon bands
well-resolved

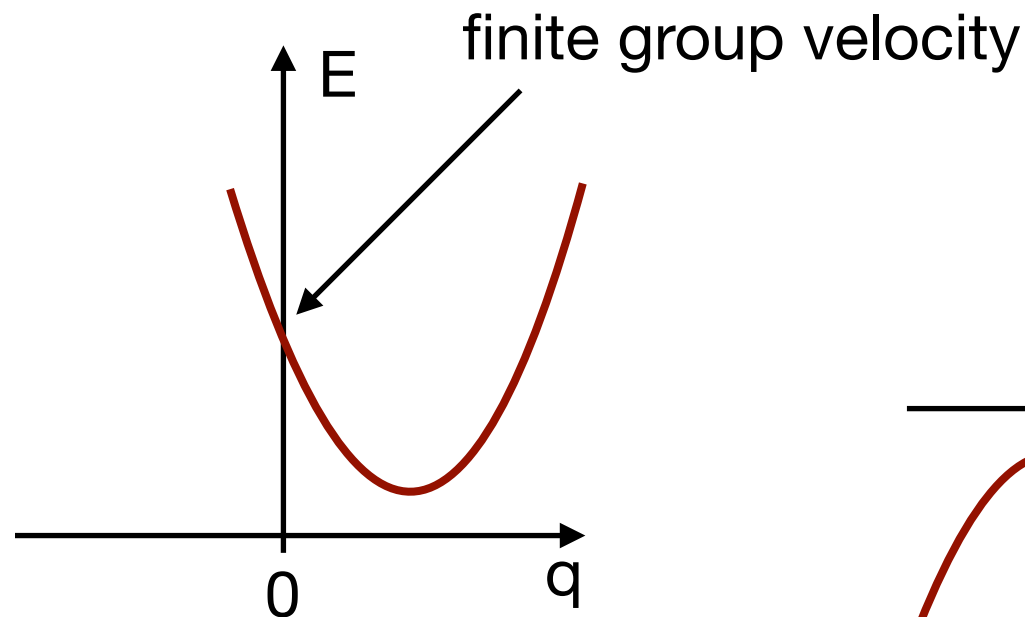
Kugler, MG *et al* PRL (2015)

Non-reciprocal magnon dynamics

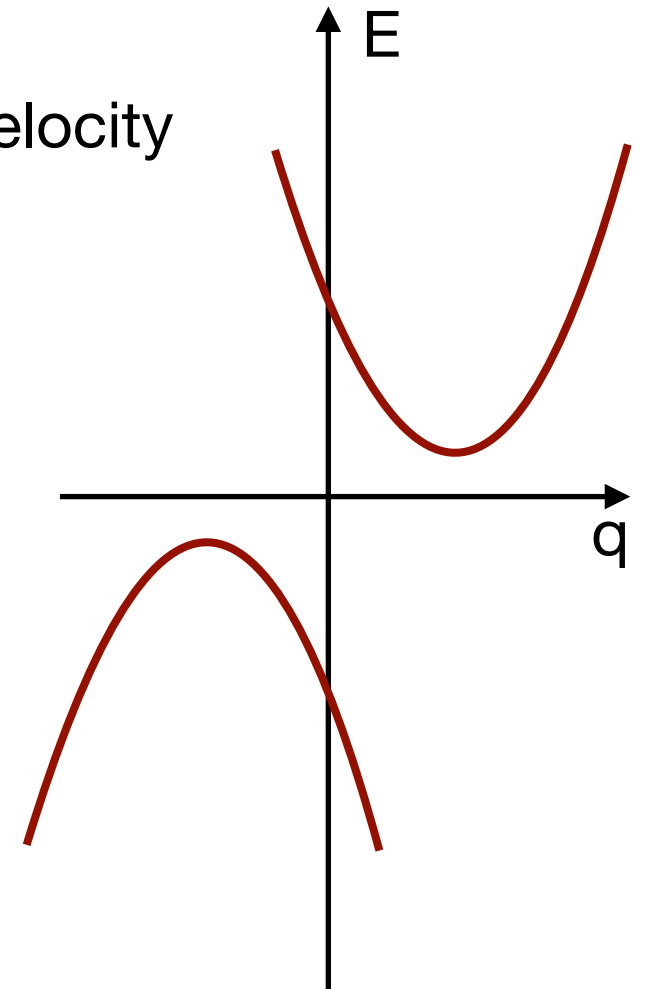
magnon dispersion in the background of field-polarised state:



without
Dzyaloshinskii-Moriya
interaction (DMI)



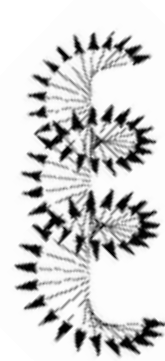
with DMI:
shifted parabola
non-reciprocal dispersion
 $E(q) \neq E(-q)$



magnon emission and
absorption at different
energies

Magnon spectrum as a function of magnetic field

magnetic state



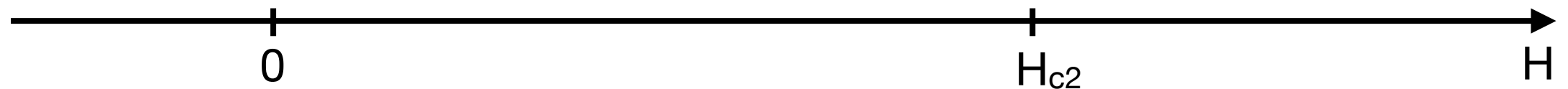
helix



conical helix

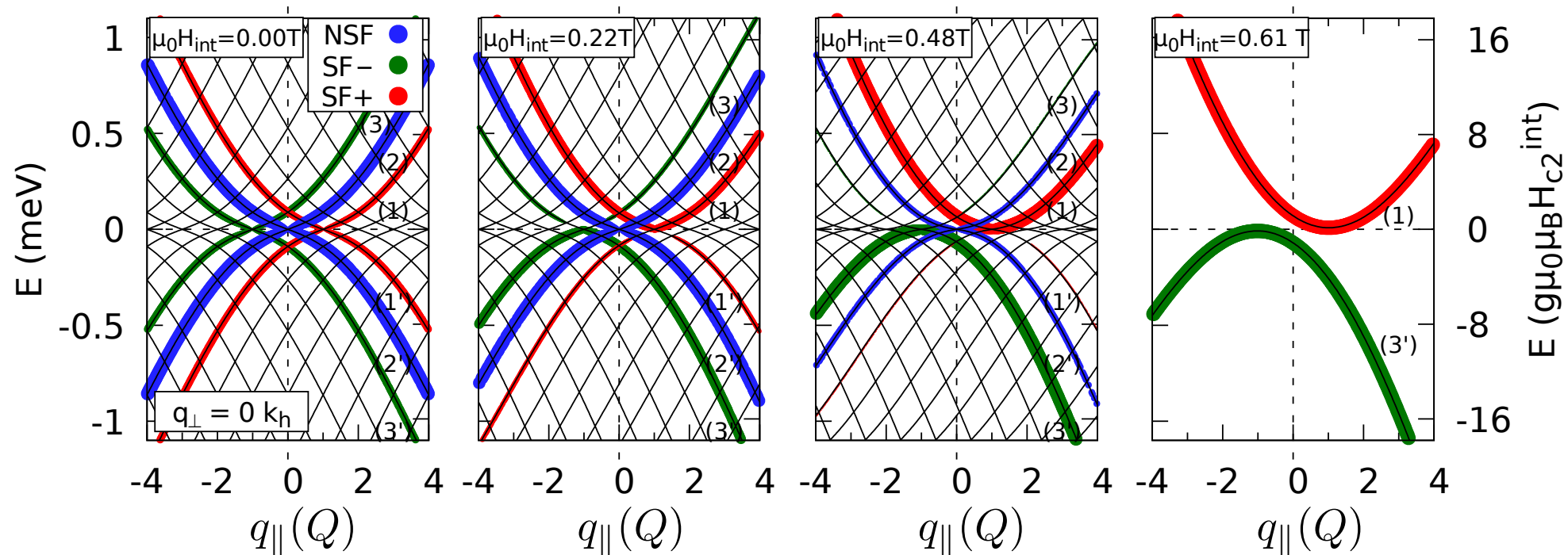


field-polarised



magnon spectrum for $q_{\perp} = 0$

- spin-flip channels
- channels
- non spin-flip channel



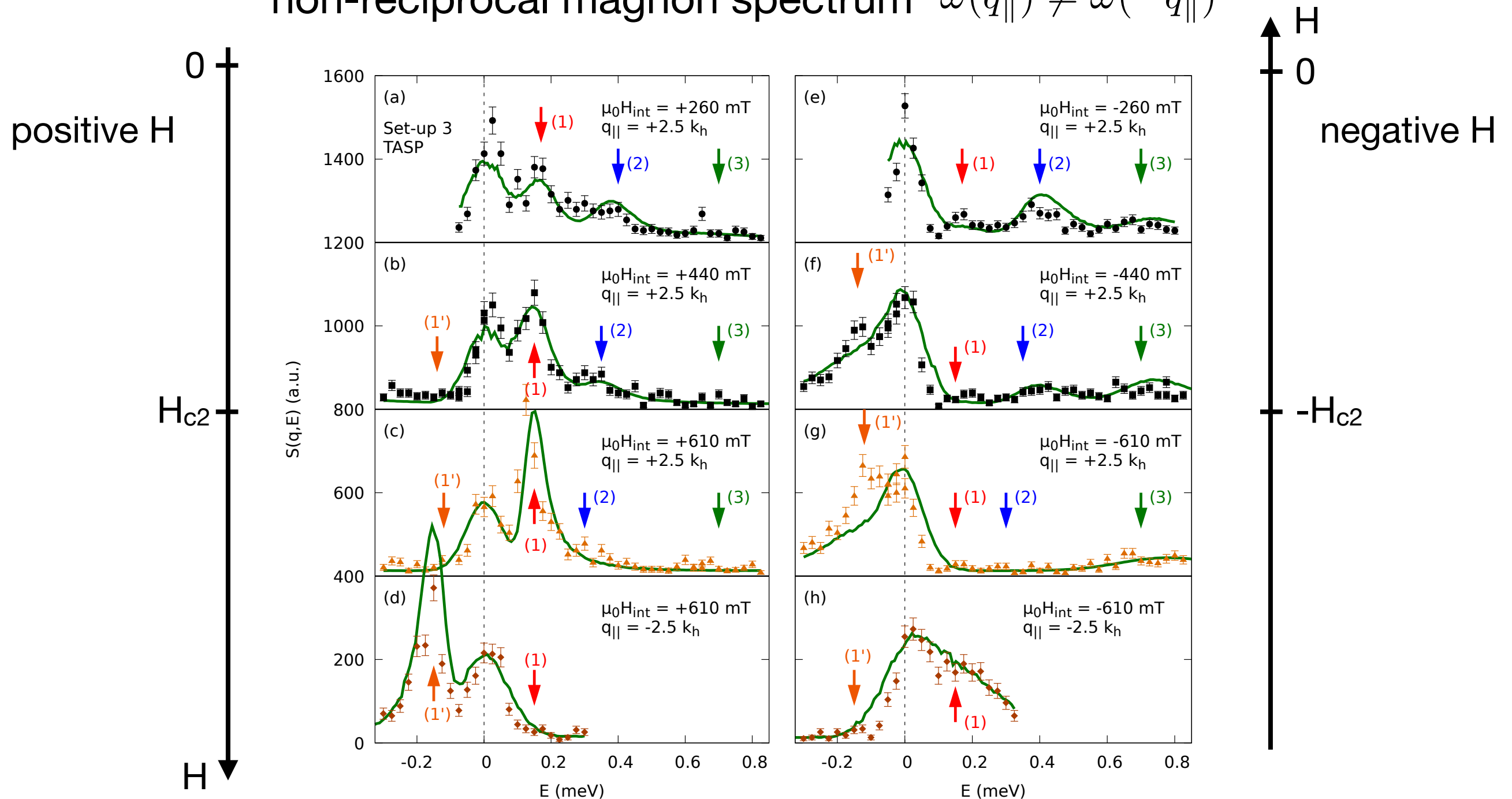
reciprocal spectrum but non-reciprocal weight distribution

non-reciprocal spectrum

Weber, MG et al arXiv:1708.02098

Inelastic neutron scattering on MnSi

non-reciprocal magnon spectrum $\omega(q_{\parallel}) \neq \omega(-q_{\parallel})$



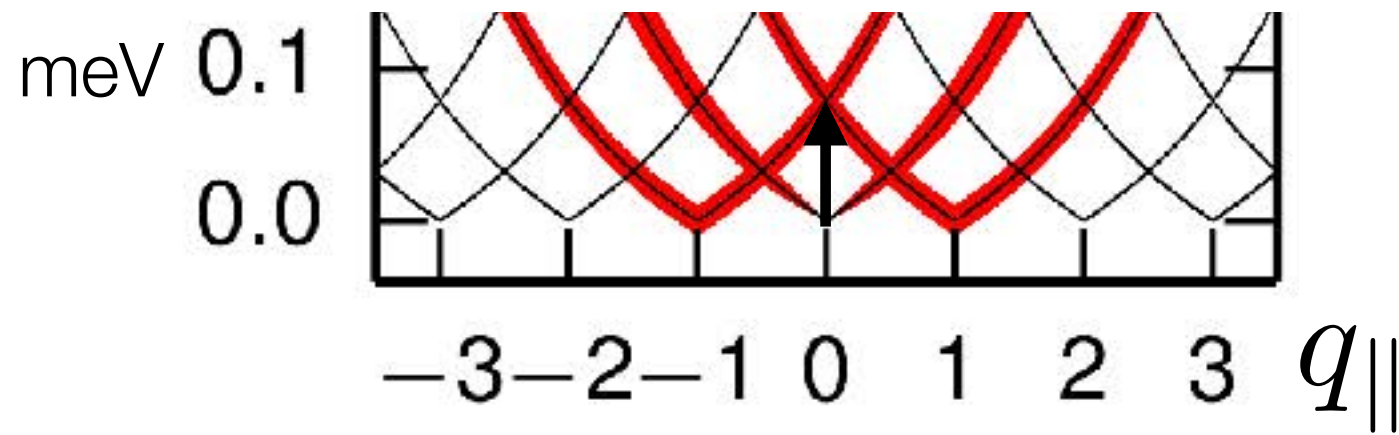
quantitative agreement between theory and experiment
(including instrumental resolution)

Weber, MG *et al*
arXiv:1708.02098

Magnetic microwave resonances

ac magnetic field \Rightarrow exciting magnons at zero momentum

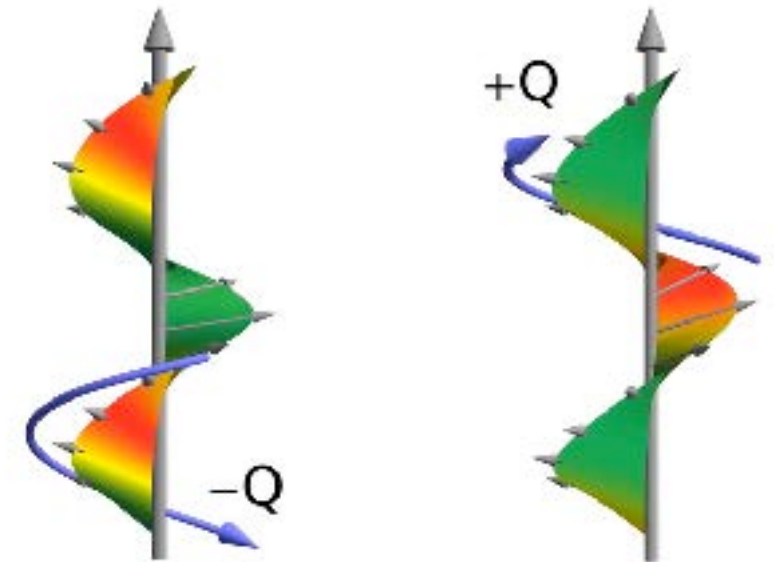
two resonances:



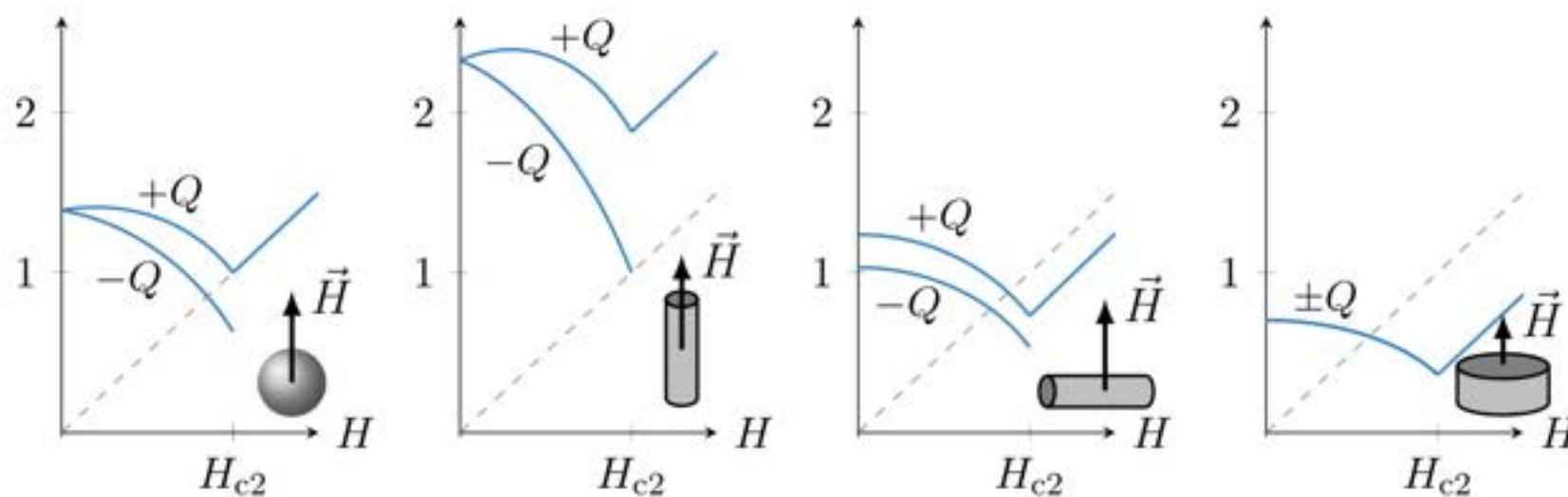
mean magnetisation oscillates

clockwise

counter-clockwise



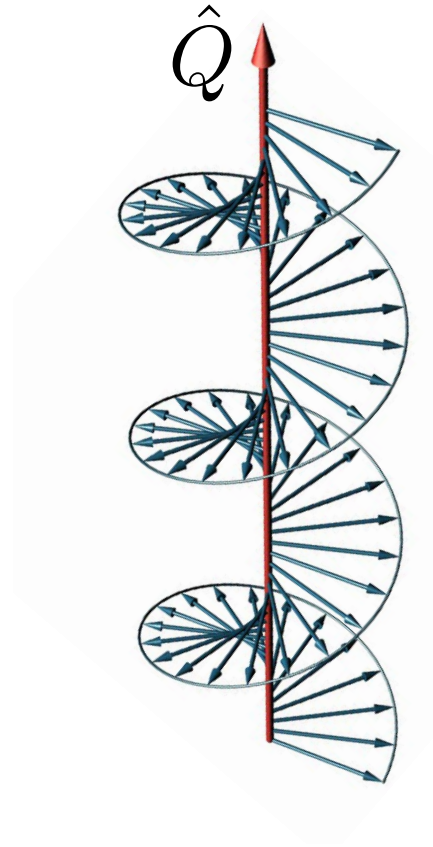
Demagnetization field splits the degeneracy:



Schwarze, MG *et al* Nat Mater (2015)

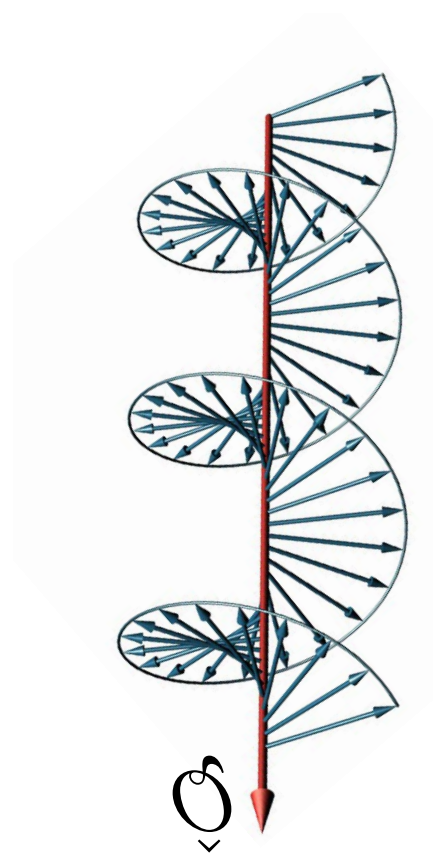
Polarization dependence of the helix modes

at zero field: helix possesses π -rotation symmetry



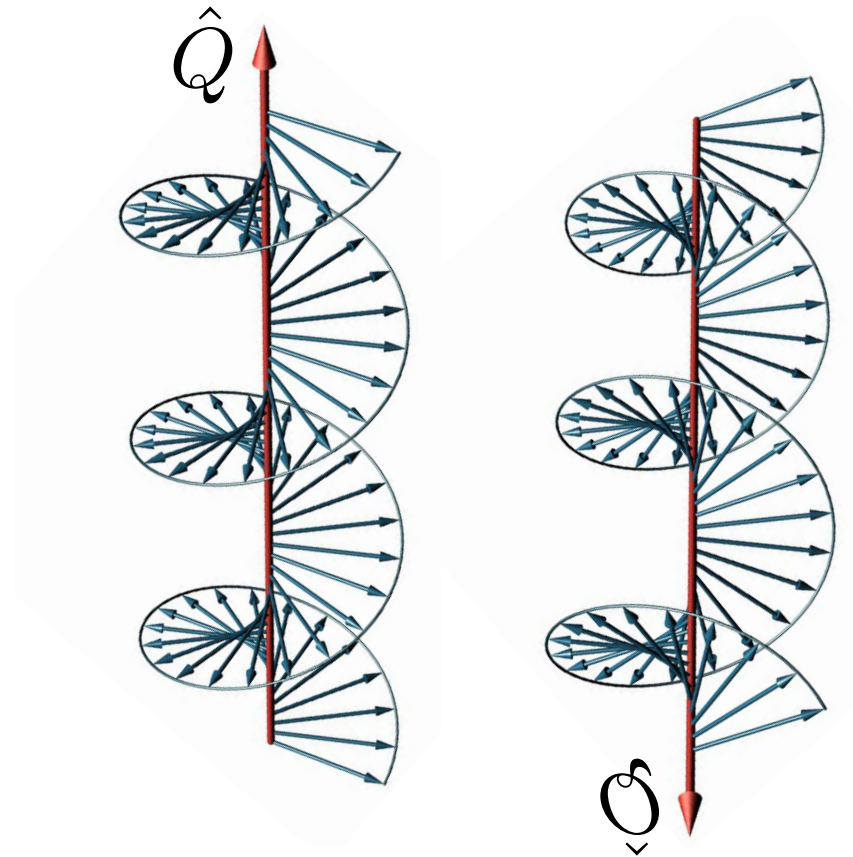
Polarization dependence of the helix modes

at zero field: helix possesses π -rotation symmetry



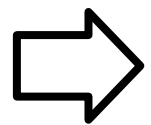
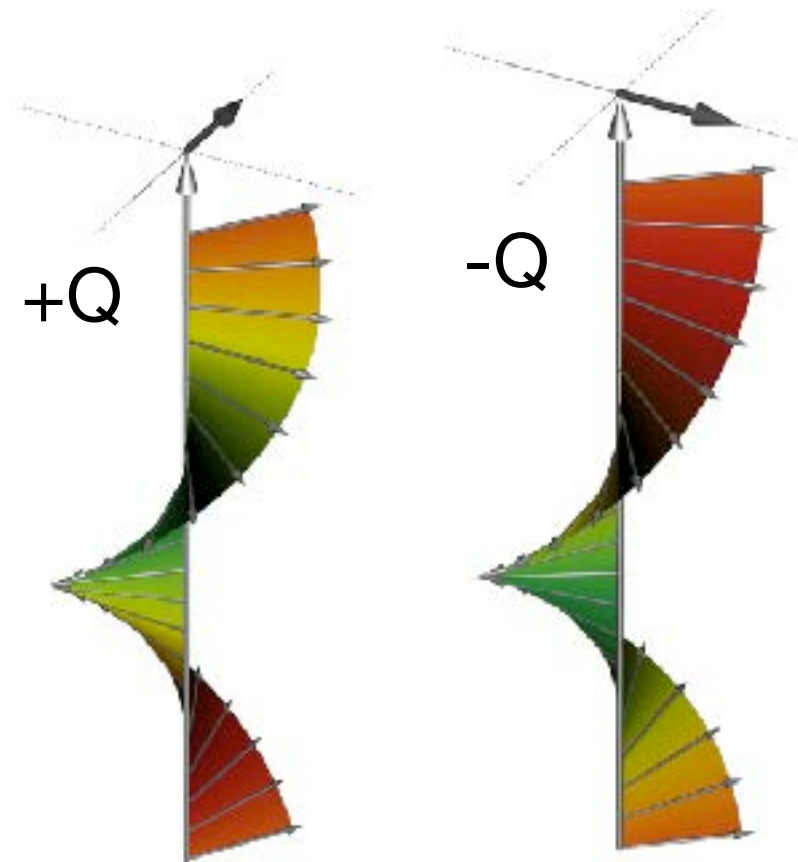
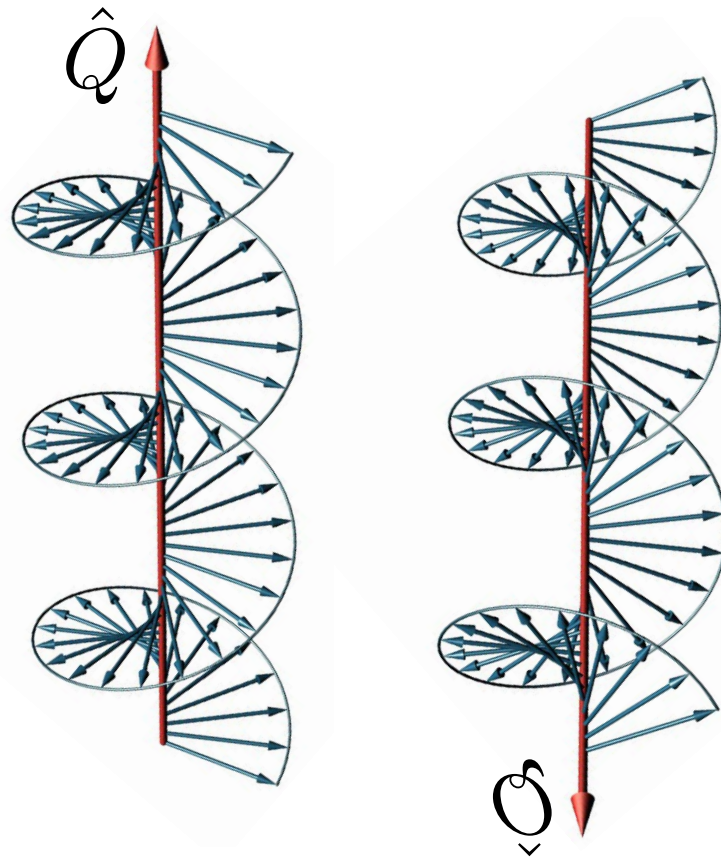
Polarization dependence of the helix modes

at zero field: helix possesses π -rotation symmetry



Polarization dependence of the helix modes

at zero field: helix possesses π -rotation symmetry



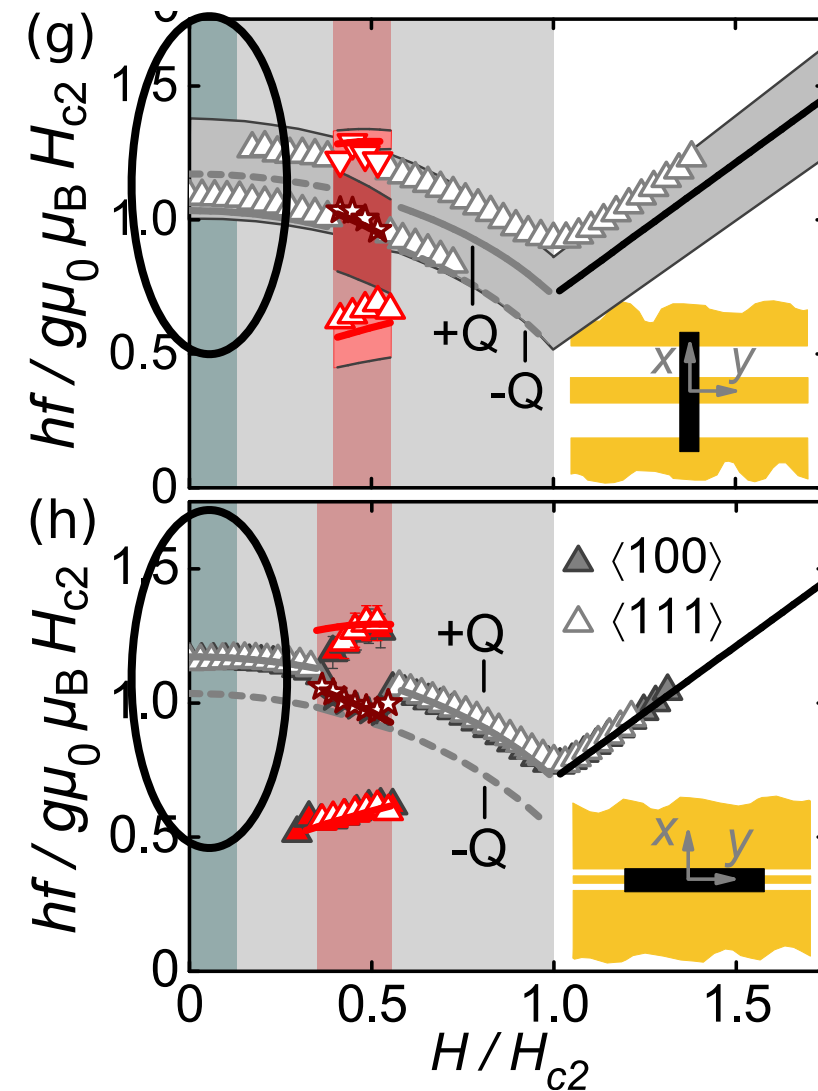
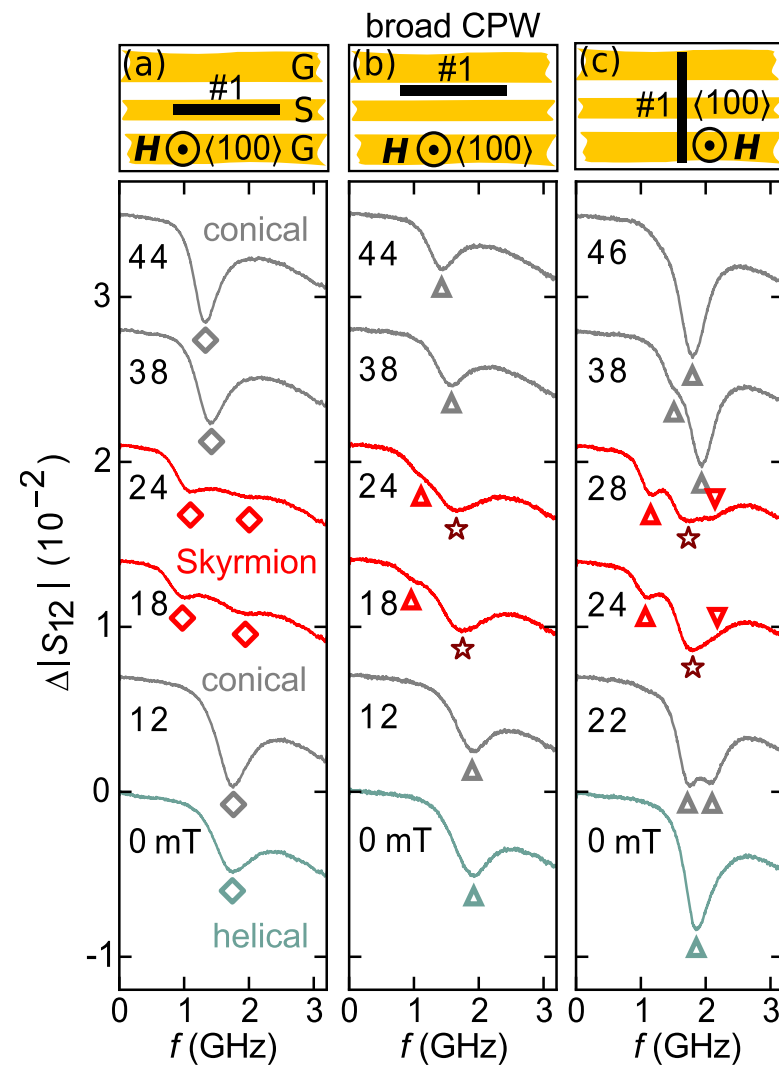
$\pm Q$ modes strictly linearly polarized at zero field!
(for non-circular sample shape)

similar to easy-plane antiferromagnets

Polarization dependence of the helix modes

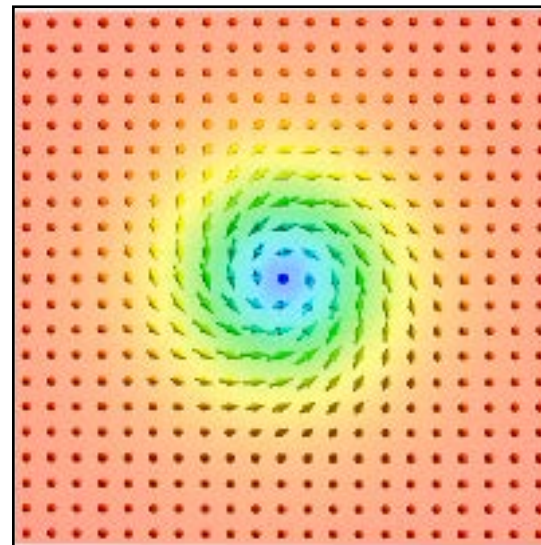
+/- Q modes strictly linearly polarized at zero field!

allows to address each mode selectively



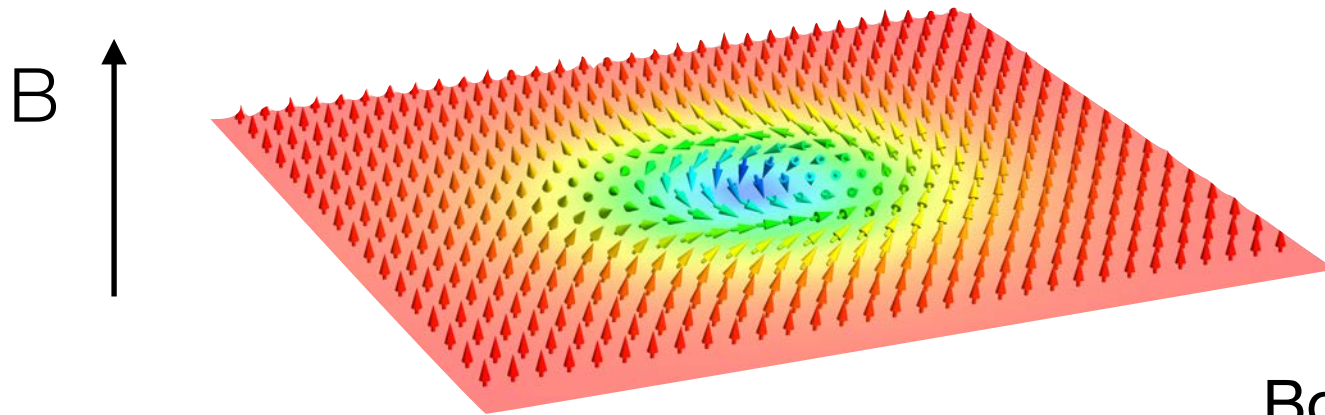
I. Stasinopoulos, MG, et al,
Scientific Reports (2017)

Spin-wave dynamics of the magnetic skyrmion



Skyrmion in a field-polarised background

static skyrmion-soliton solution



Bogdanov & Hubert (1994)

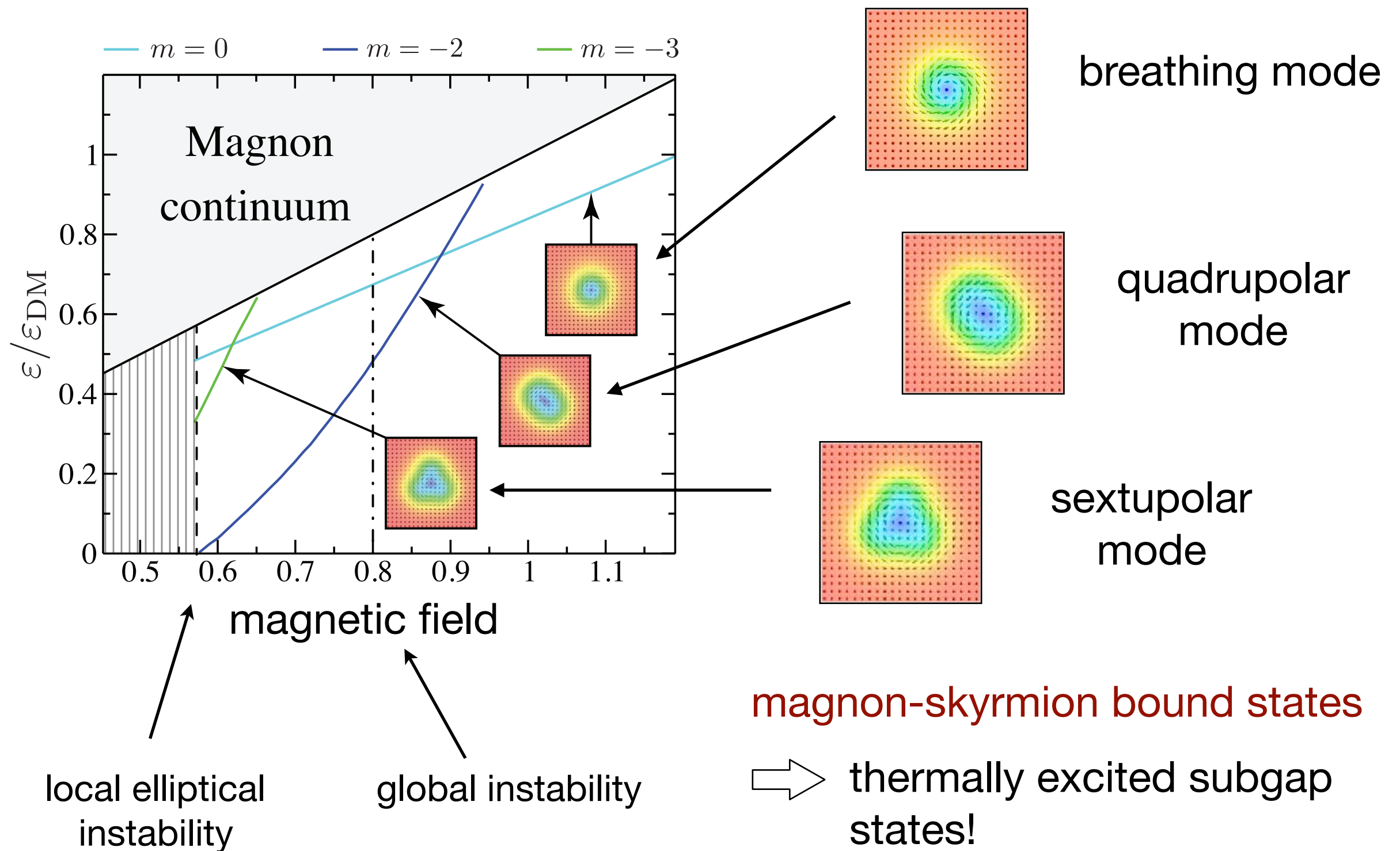
spin-waves scatter off the skyrmion \Rightarrow magnon scattering problem

magnon Hamiltonian

$$\mathcal{H} = \frac{\hbar^2 (-i\tau^z \vec{\nabla} - \mathbf{1}\vec{a})^2}{2M_{\text{mag}}} + \mathbf{1}\mathcal{V}_0 + \tau^x \mathcal{V}_x$$

\nearrow
scattering vector
potential \uparrow \uparrow
scattering potentials

Magnon-skyrmion bound states



see also Lin, Batista & Saxena PRB (2014)

Schütte & MG PRB (2014)

Emergent magnon Lorentz force

adiabatic adjustment of local frame \Rightarrow Berry phase

vector scattering potential

$$\vec{a} = \left(\frac{\cos \theta}{\rho} - Q \sin \theta \right) (-\sin \chi, \cos \chi)$$

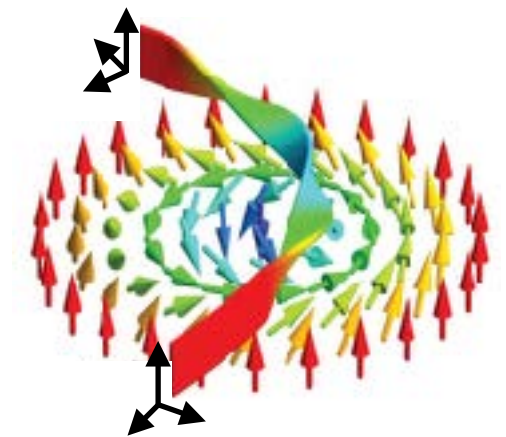
with quantised total flux

$$\int d^2\mathbf{r} (\nabla \times \vec{a}) = \int d^2\mathbf{r} \hat{M} (\partial_x \hat{M} \times \partial_y \hat{M})$$

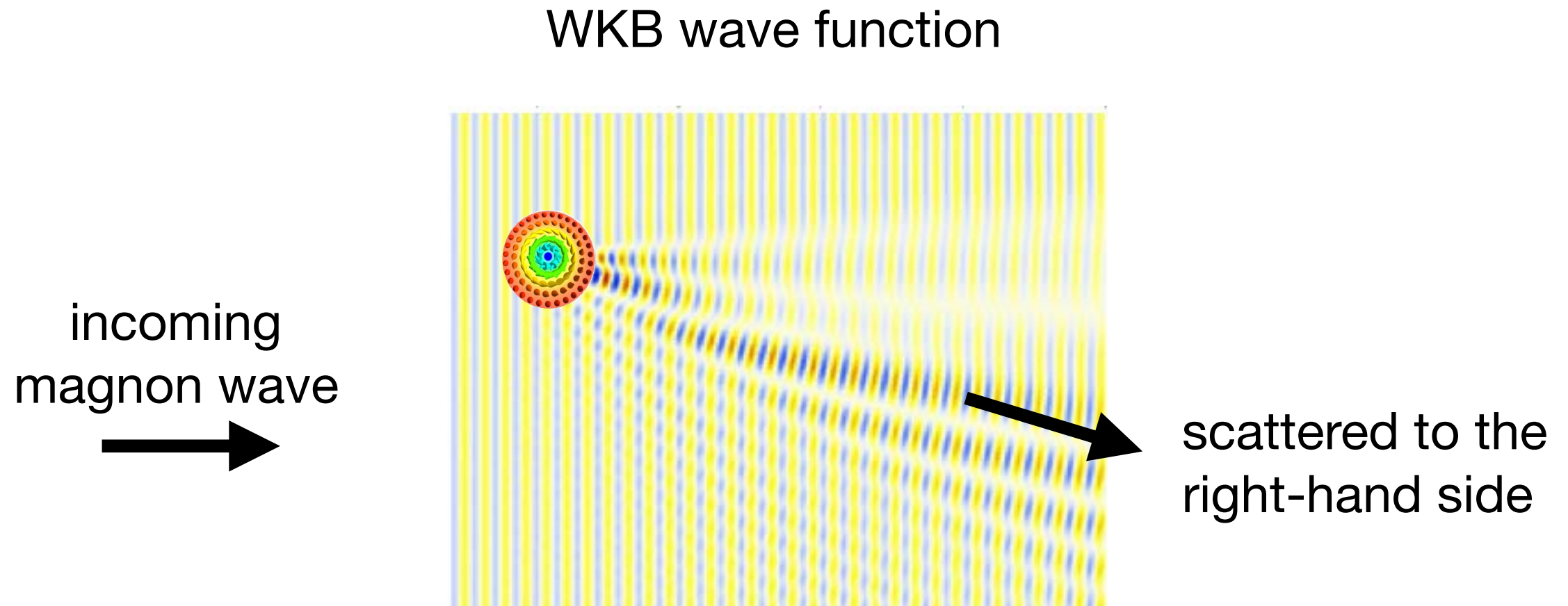
\Leftarrow topological winding number

magnon scatter off a localised emergent magnetic flux
due to non-trivial topology of skyrmion

\Rightarrow emergent Lorentz force



Topological magnon skew scattering



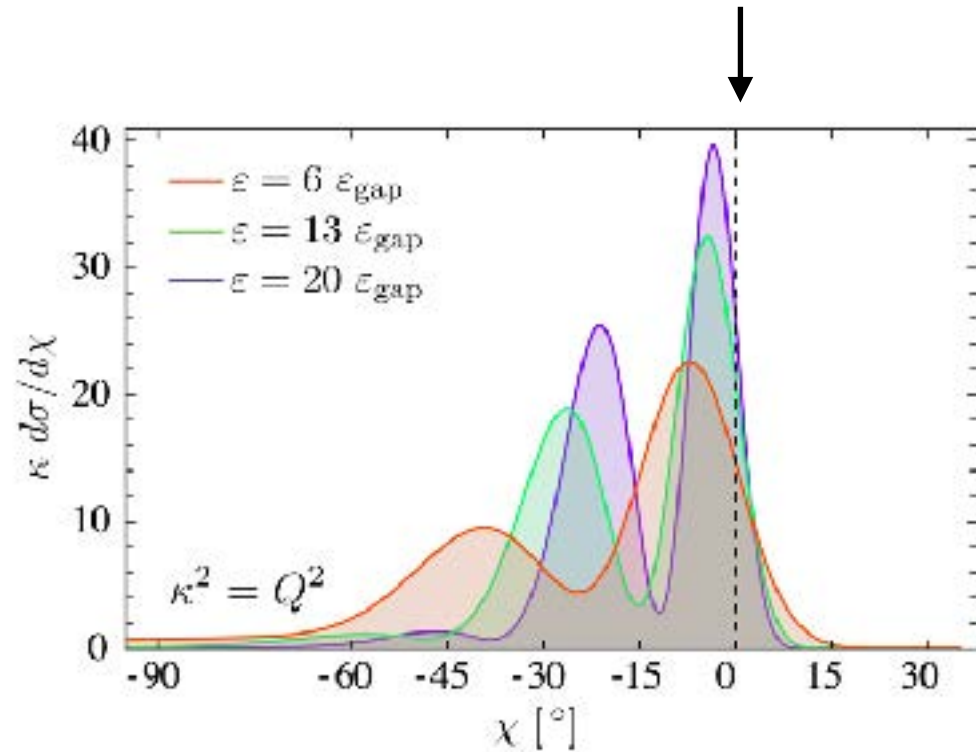
emergent Lorentz force leads to **skew scattering!**

⇒ topological magnon Hall effect!

see also Iwasaki, Beekman & Nagaosa PRB (2014)
Mochizuki *et al.* Nat. Mat. (2014)

Schütte & MG PRB (2014)
Schroeter & MG LTP (2015)

Skew & rainbow scattering

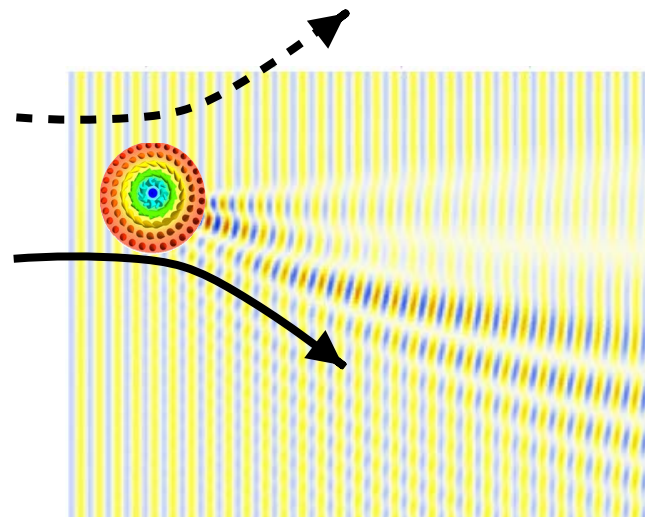


magnon differential cross section

asymmetric & oscillations

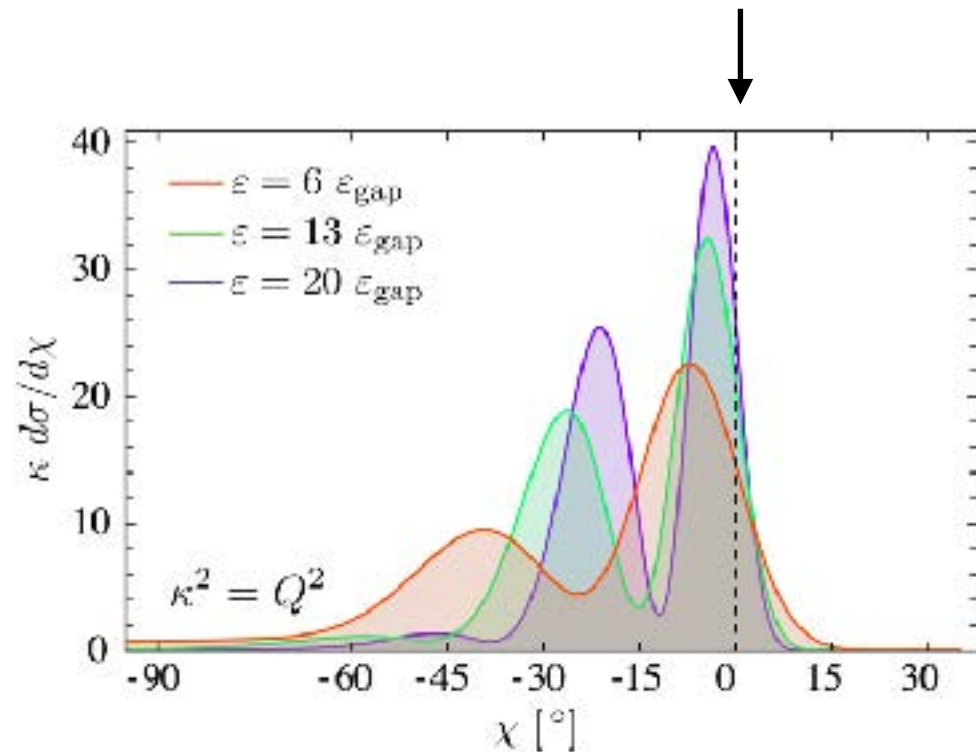
Different classical trajectories contribute and interfere!

rainbow scattering!



Schütte & MG PRB (2014)
Schroeter & MG LTP (2015)

Skew & rainbow scattering

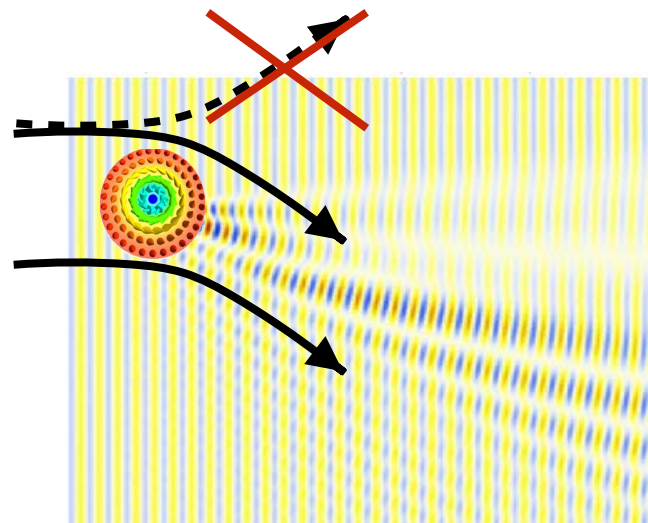


magnon differential cross section

asymmetric & oscillations

Different classical trajectories contribute and interfere!

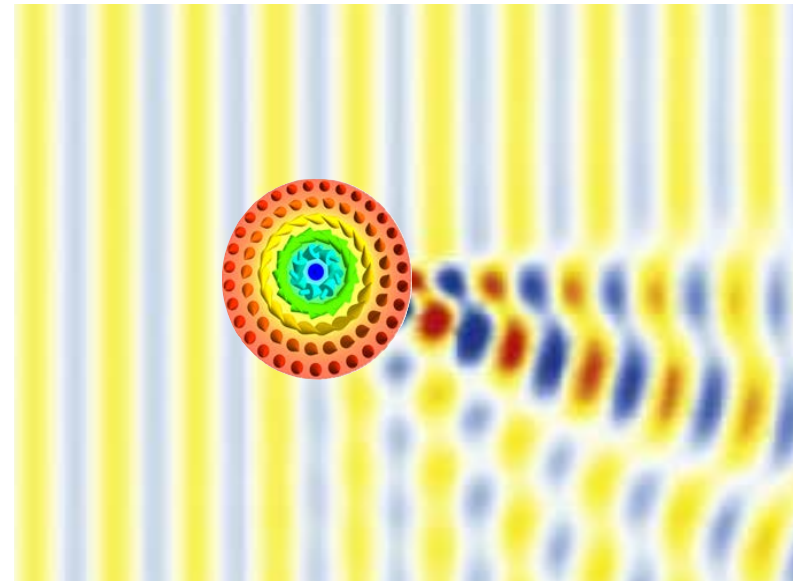
rainbow scattering!



Schütte & MG PRB (2014)
Schroeter & MG LTP (2015)

How to drive skyrmions with magnon currents?

incoming
momentum



magnon wave exerts a pressure on the skyrmion...

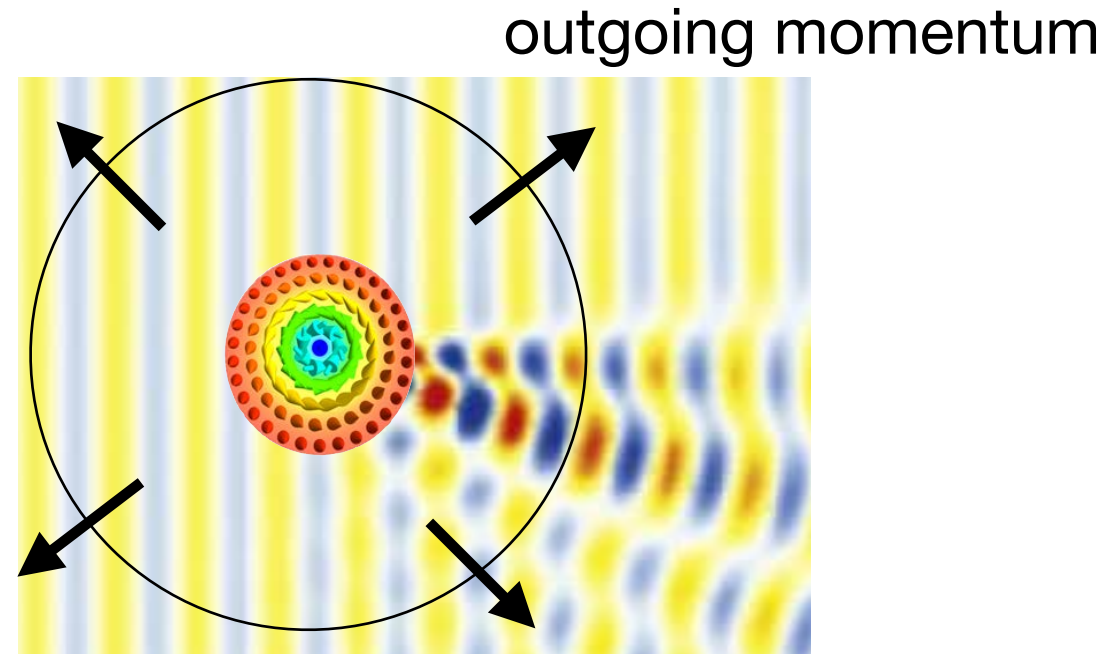
in which direction will it move?

momentum conservation?

Driving skyrmions with magnon currents

counting **net flux of incoming
& outgoing momentum**

incoming
momentum



conservation law of linear momentum

$$4\pi\mathcal{S}\epsilon_{0\mu\nu}j_{\mu}^{\text{top}} + \partial_{\mu}T_{\mu\nu}^{\text{stat}} = 0$$

space-time topological current

$$j_{\mu}^{\text{top}} = \frac{1}{8\pi}\epsilon_{\mu\nu\lambda}\hat{M}(\partial_{\nu}\hat{M} \times \partial_{\lambda}\hat{M})$$

energy-momentum tensor
(static part)

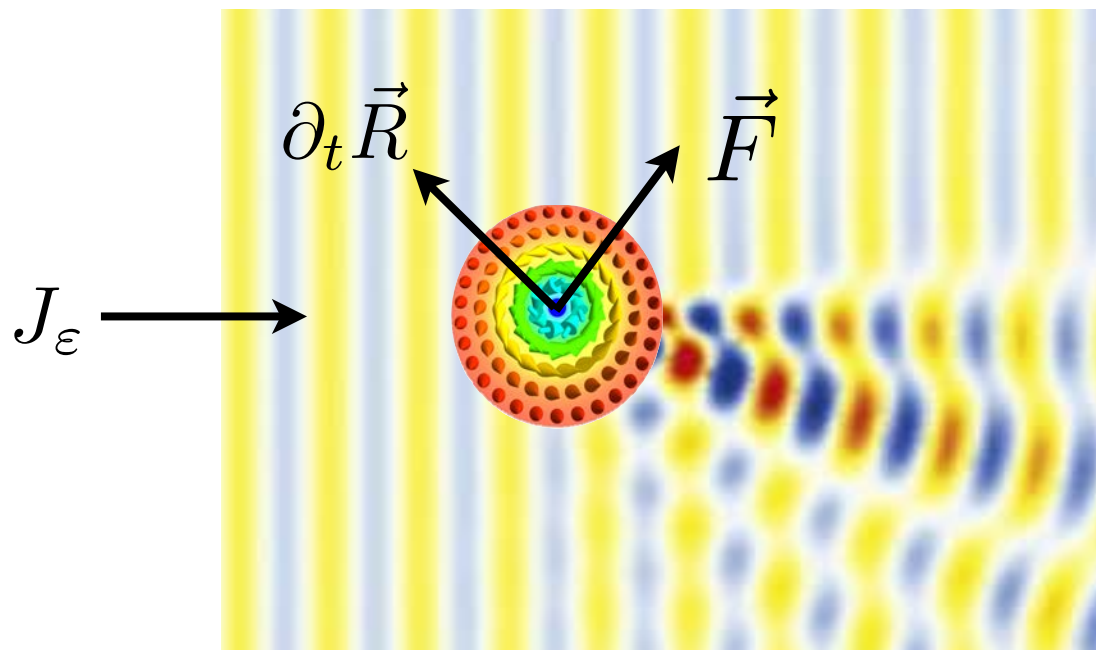
expanding conservation law up to quadratic order in ψ and integrating over the space

Linear response approximation

Thiele equation with a magnon force

$$\vec{G} \times \partial_t \vec{R}(t) = \vec{F}$$

Linear response: evaluate force \vec{F} for skyrmion at rest $\dot{\vec{R}} = 0$



momentum-transfer force

after some algebra
using optical theorem:

$$\vec{F} = J_\epsilon k \begin{pmatrix} \sigma_{\parallel}(\epsilon) \\ \sigma_{\perp}(\epsilon) \end{pmatrix}$$

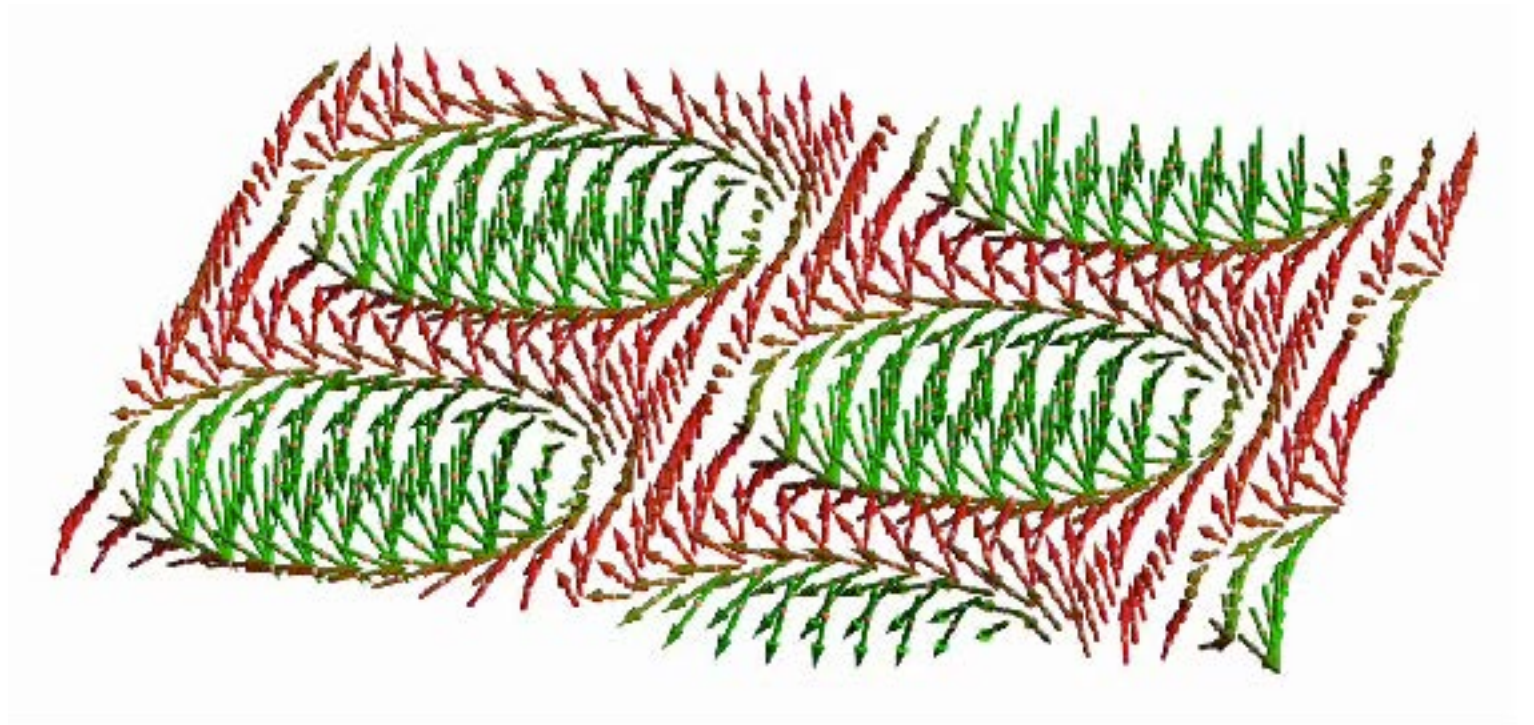
magnon force determined by
transport scattering cross sections:

with

$$\begin{pmatrix} \sigma_{\parallel}(\epsilon) \\ \sigma_{\perp}(\epsilon) \end{pmatrix} = \int_0^{2\pi} d\chi \begin{pmatrix} 1 - \cos \chi \\ -\sin \chi \end{pmatrix} \frac{d\sigma(\epsilon)}{d\chi}$$

skew scattering \rightarrow finite transversal force

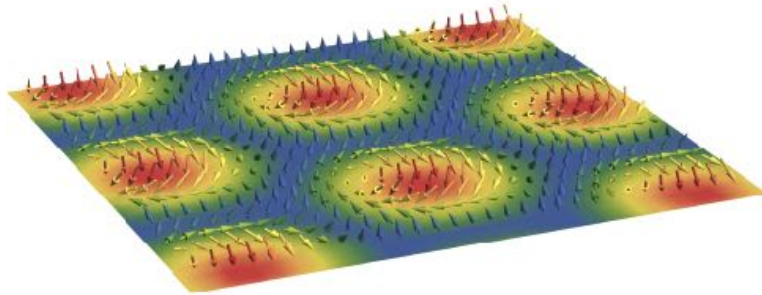
Spin-wave dynamics of the magnetic skyrmion lattice



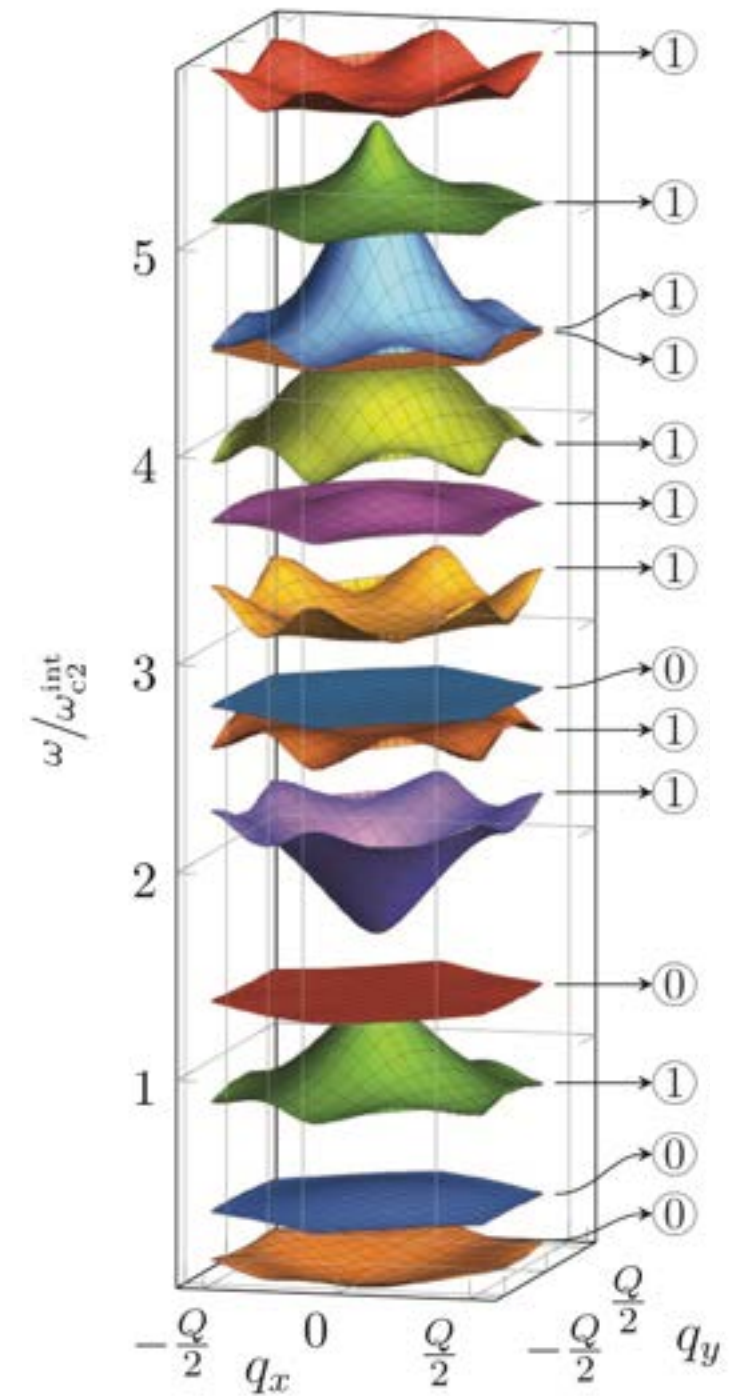
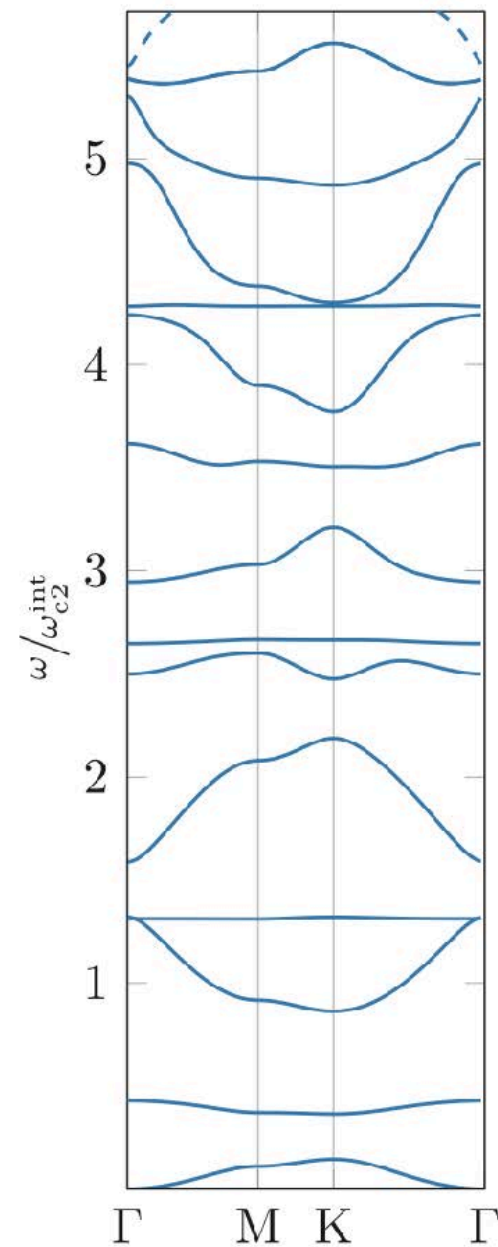
Magnon-band structure of skyrmion lattice

magnon dispersion for in-plane momenta

skyrmion lattice



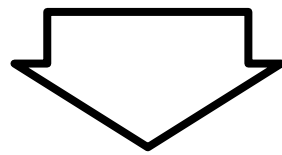
2d magnetic
Brillouin zone



Topological magnon-band structure

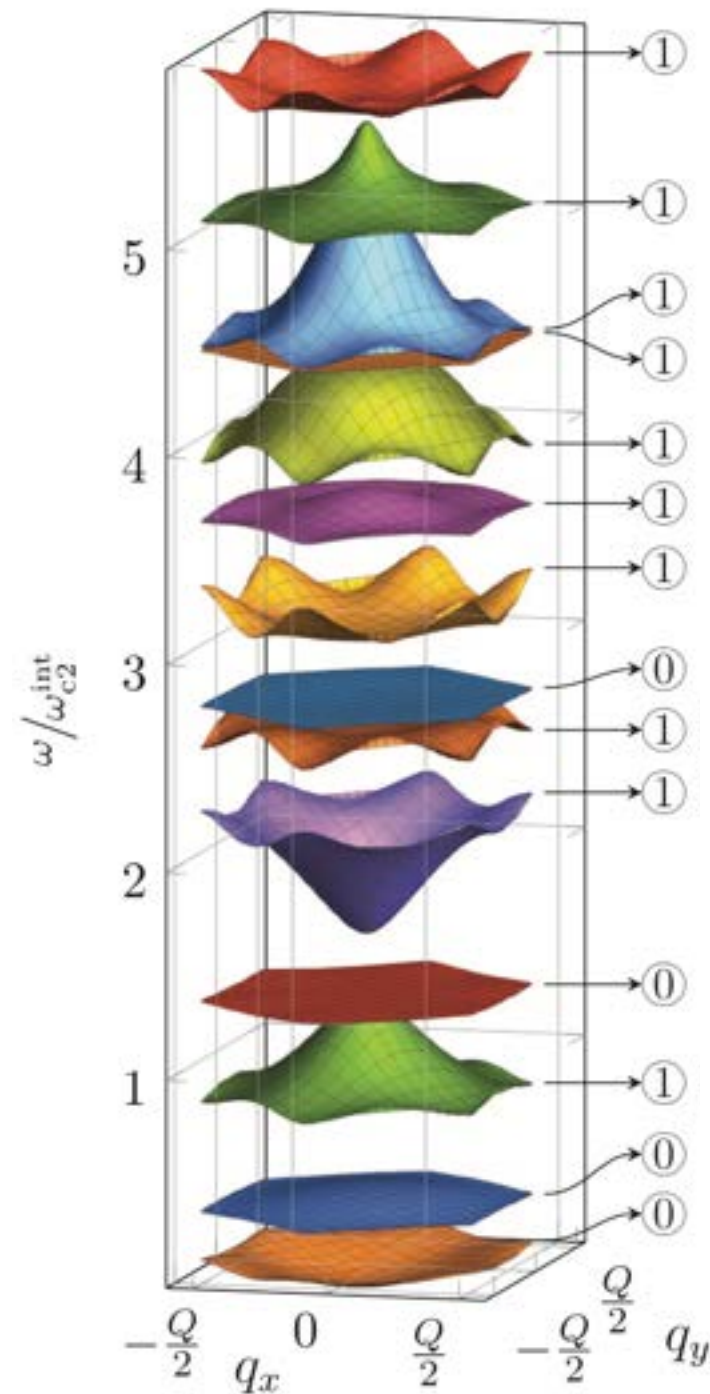
non-trivial topology of skyrmions \Rightarrow topological magnon band structure

each skyrmion acts like a source
of emergent magnetic flux

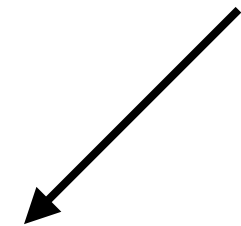


emergent magnon electrodynamics

- emergent magnon Landau levels
- bands with finite Chern numbers
- topologically protected magnon edge states

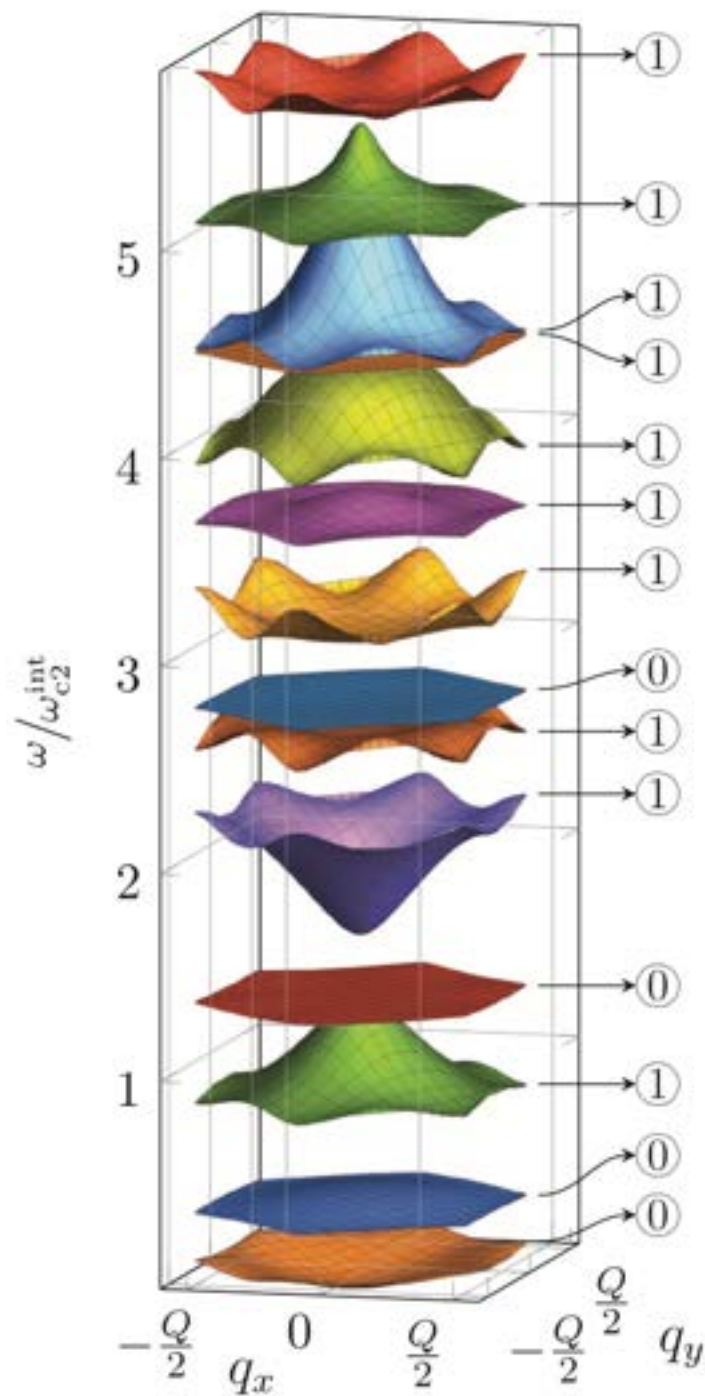


Chern numbers



Magnetic skyrmion resonances

magnetic resonances at the Γ point



clockwise
(CW)



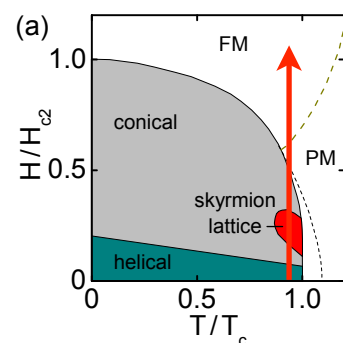
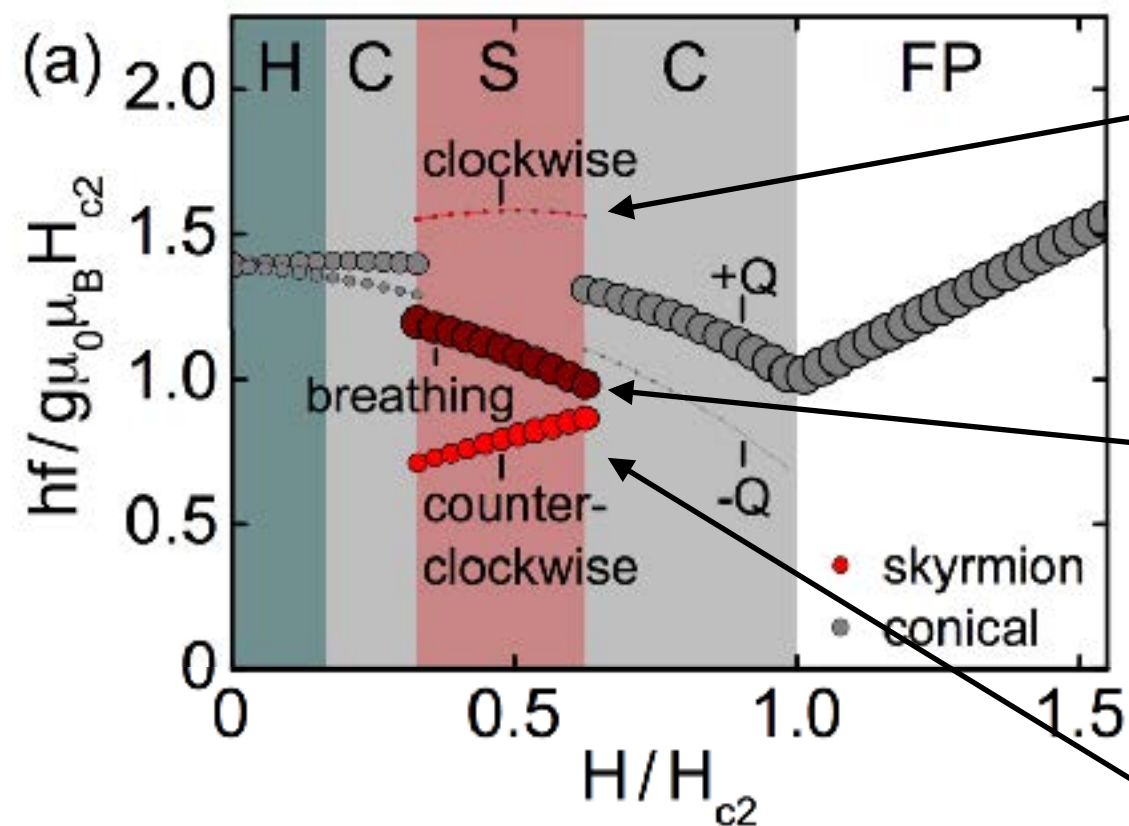
breathing



counter-
clockwise
(CCW)

Magnetic skyrmion resonances

field-dependence of the
resonance frequencies



for a field sweep
through the phase diagram



clockwise
(CW)



breathing



counter-
clockwise
(CCW)

see also Mochizuki PRL (2012);
Onose et al. PRL (2012)

Comparison experiment & theory

three different materials

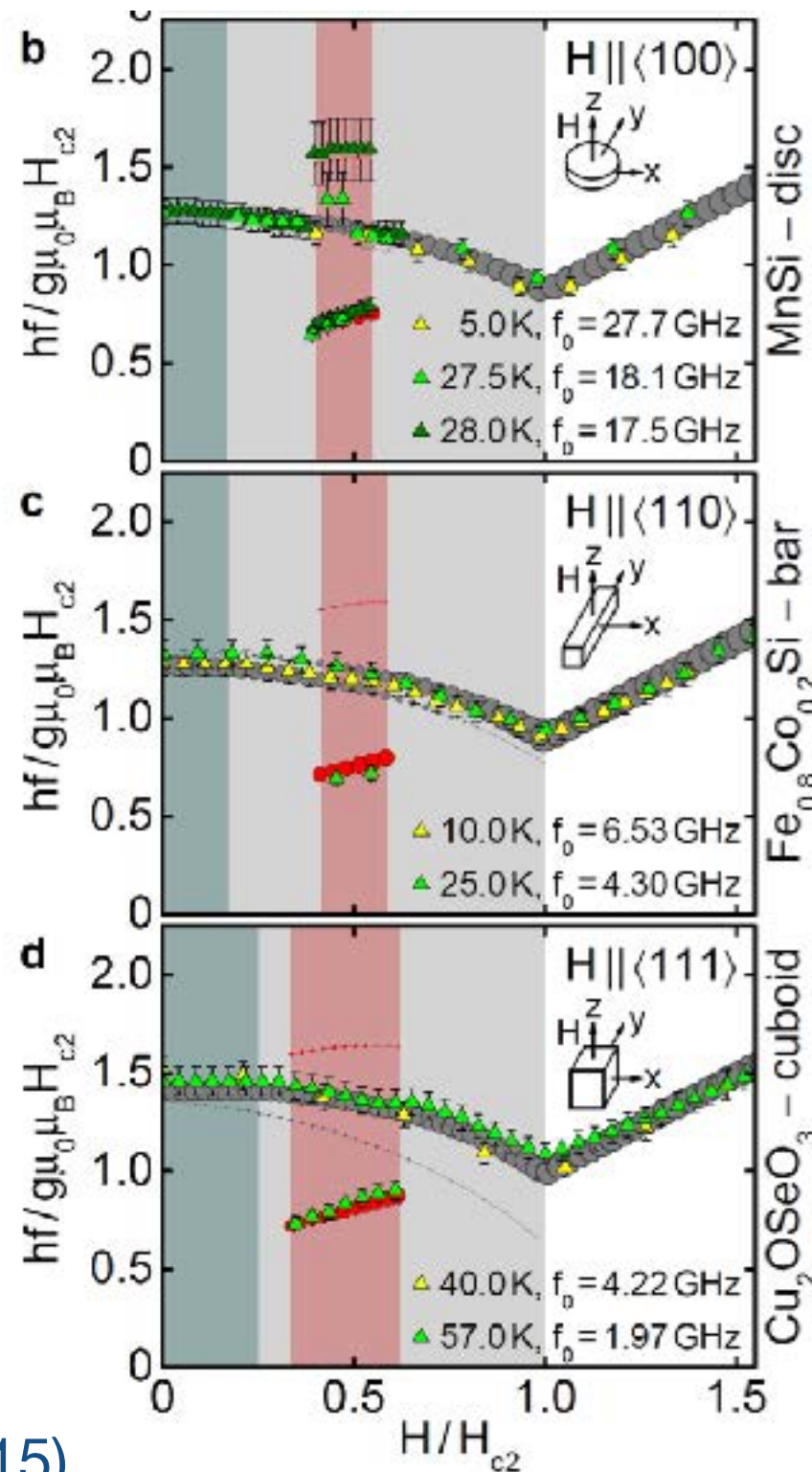
MnSi, $\text{Fe}_{0.8}\text{Co}_{0.2}\text{Si}$ and

Cu_2OSeO_3

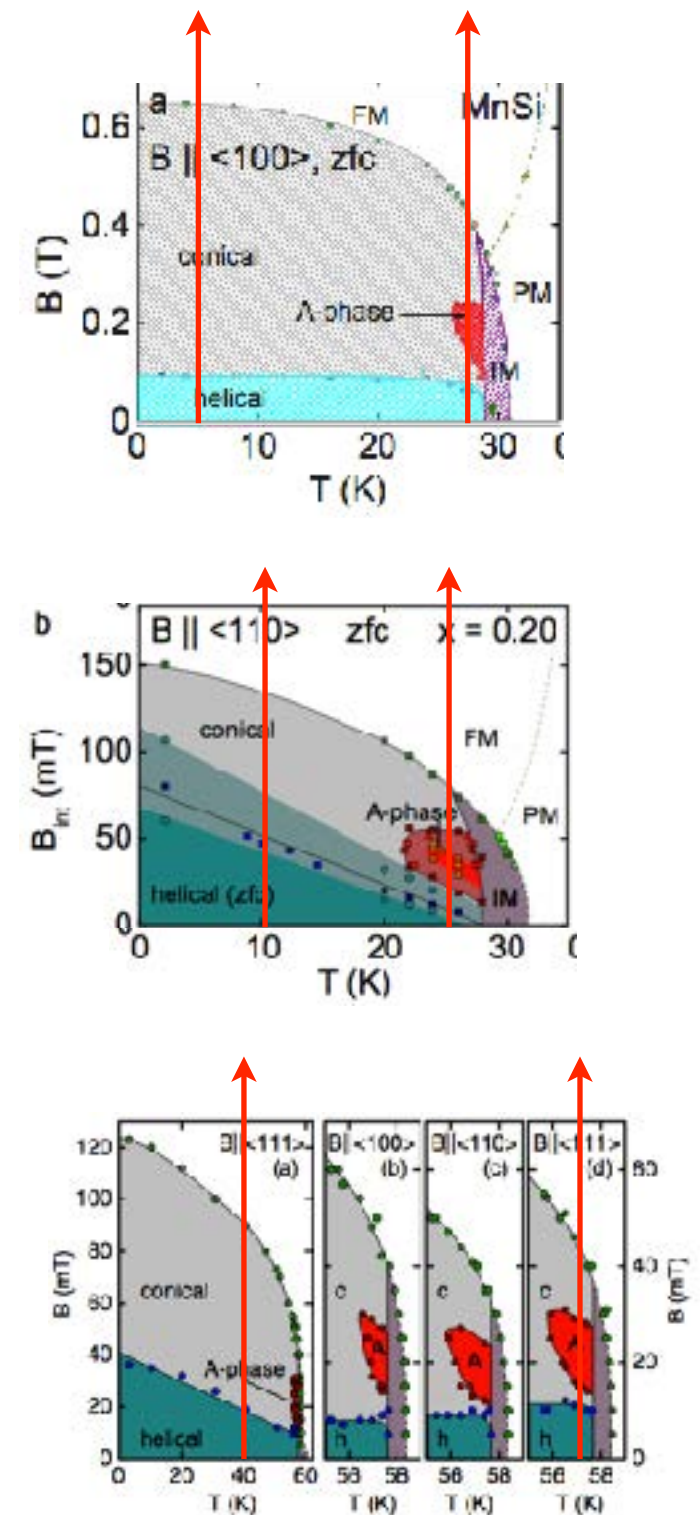
with three different shapes

(demagnetization factors)

excellent quantitative
theoretical understanding



different field sweeps
normalized with $H_{c2}(T)$

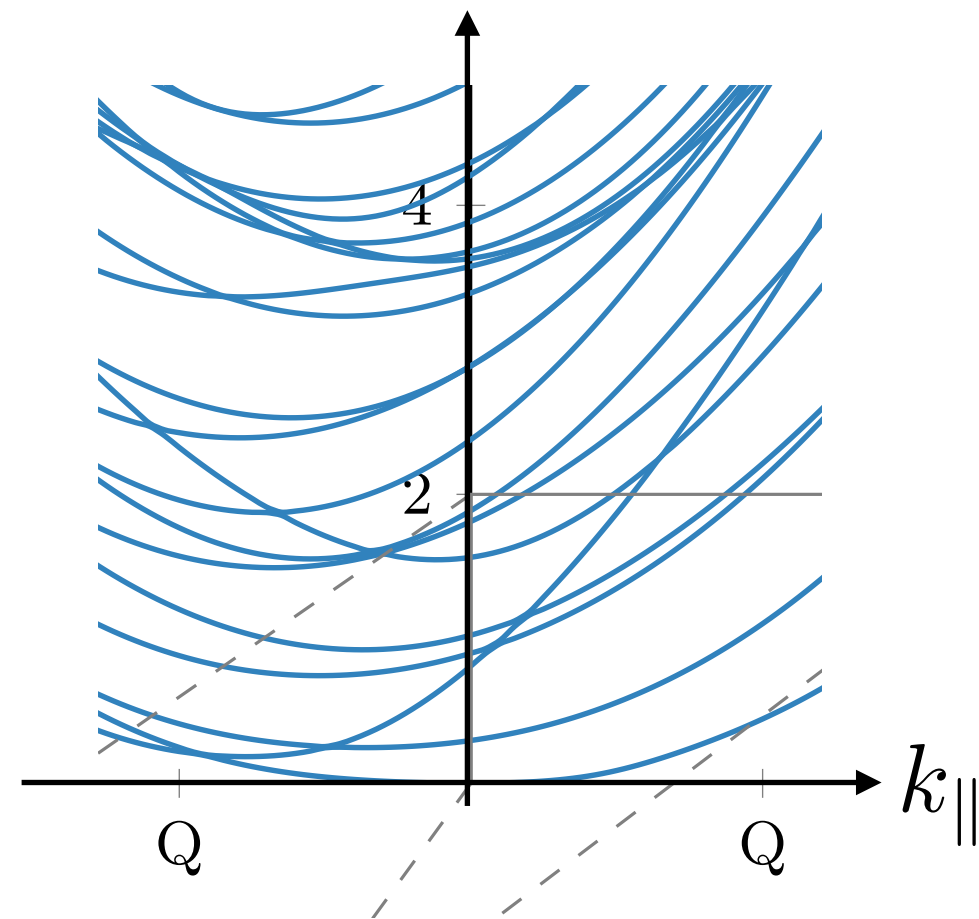
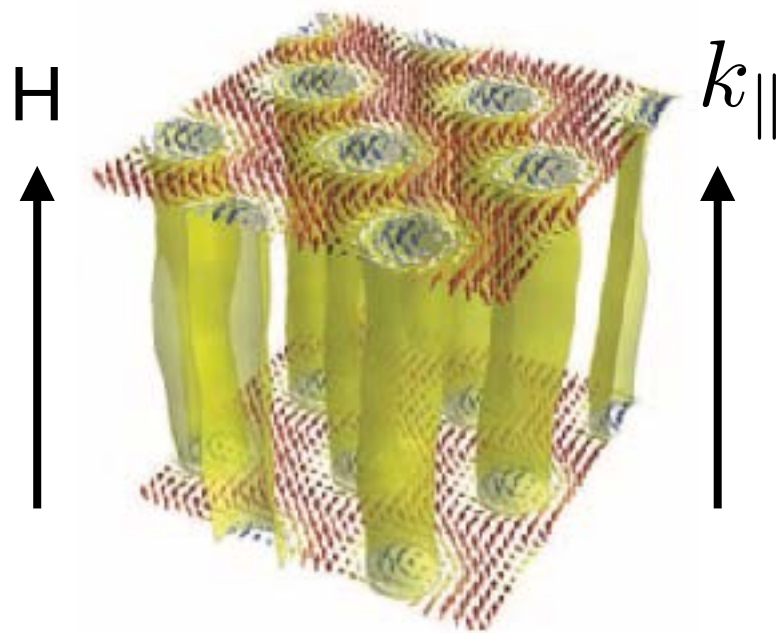


T. Schwarze, *et al.* Nat Mater (2015)

Non-reciprocity of skyrmion-lattice magnons

magnon dispersion for out-of-plane momenta

skyrmion lattice:

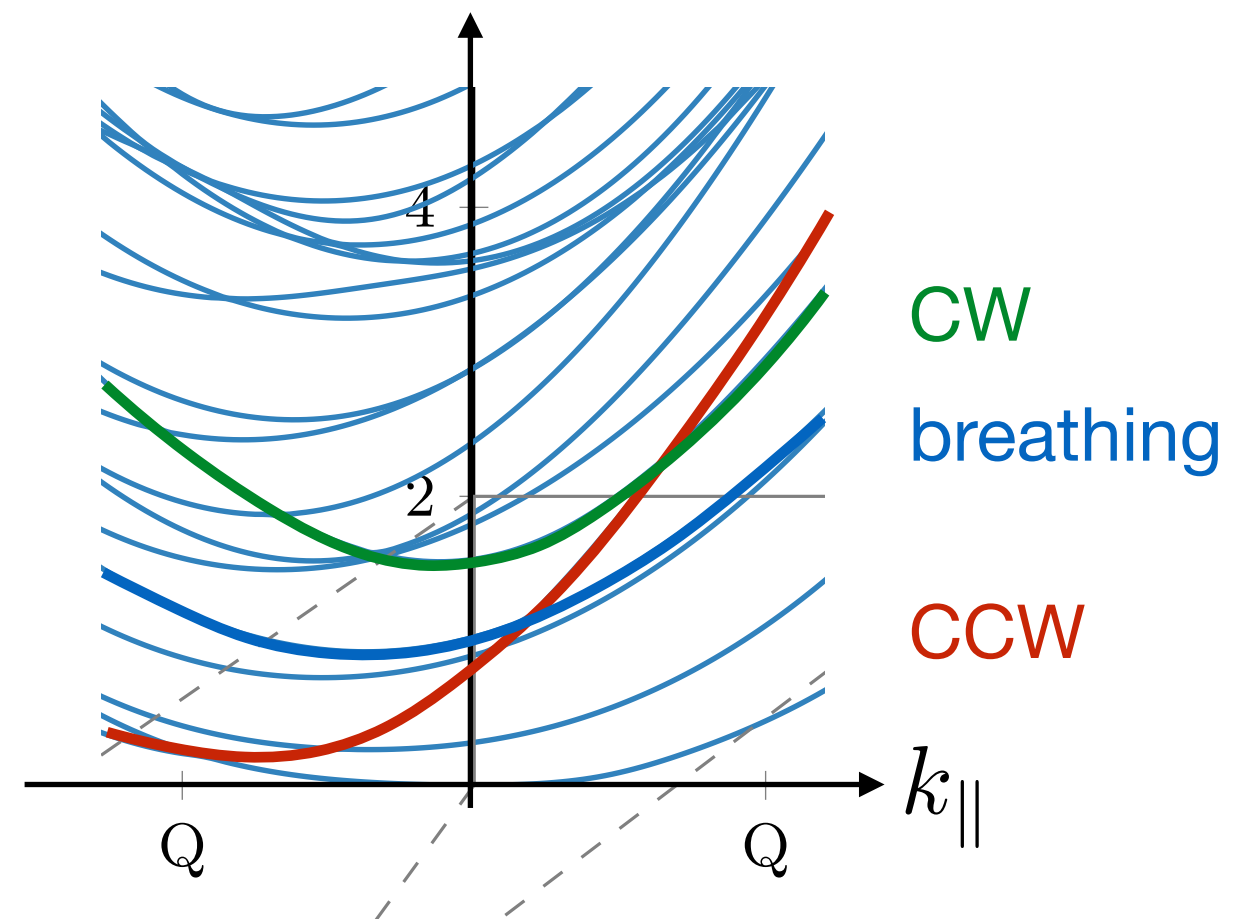
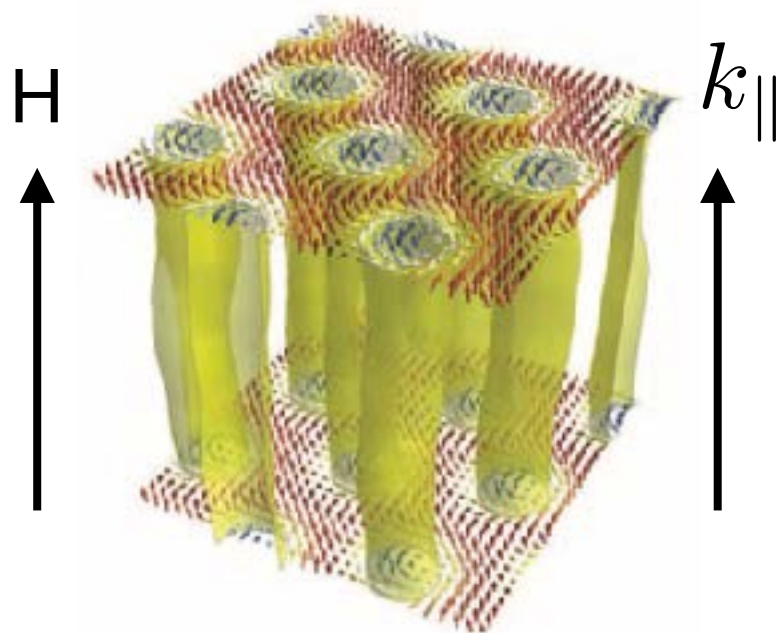


non-reciprocity $\omega(k_{||}) \neq \omega(-k_{||})$

Non-reciprocity of skyrmion-lattice magnons

magnon dispersion for out-of-plane momenta

skyrmion lattice:



non-reciprocity $\omega(k_{||}) \neq \omega(-k_{||})$

most pronounced for the CCW mode!

confirmed by spin-wave spectroscopy experiments by S. Seki!

Non-linear excitations...

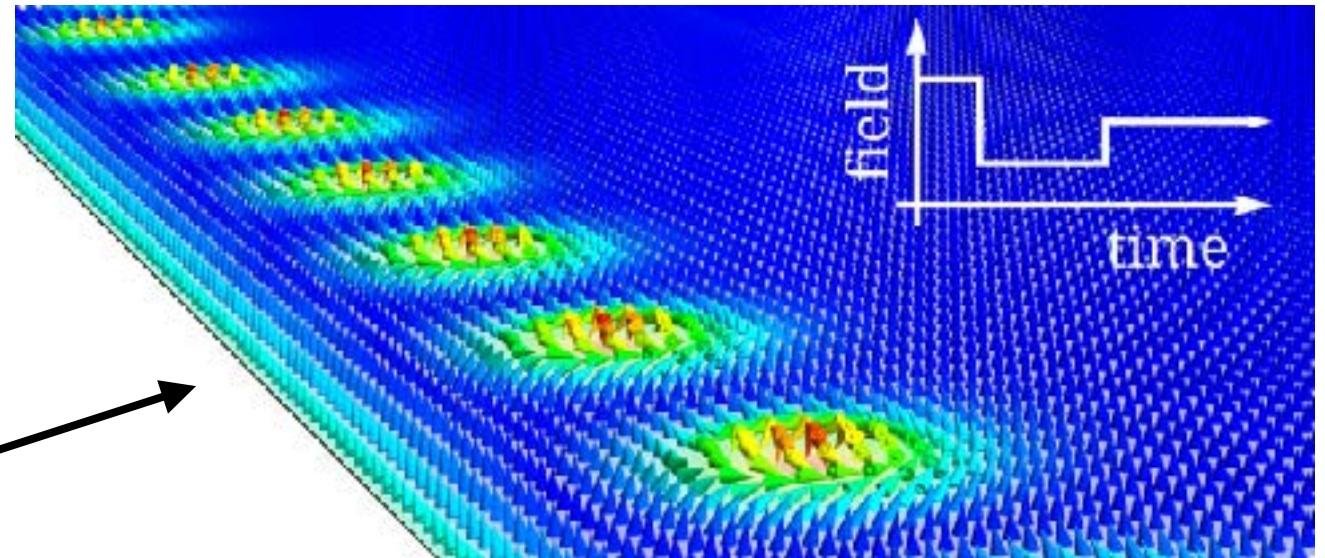
Skyrmion creation at the edge

uncompensated Dzyaloshinskii-Moriya interaction at the edge:

boundary condition:

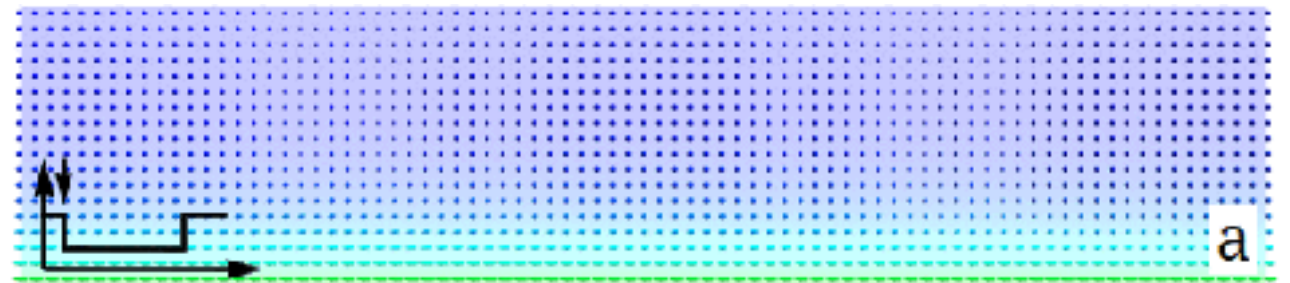
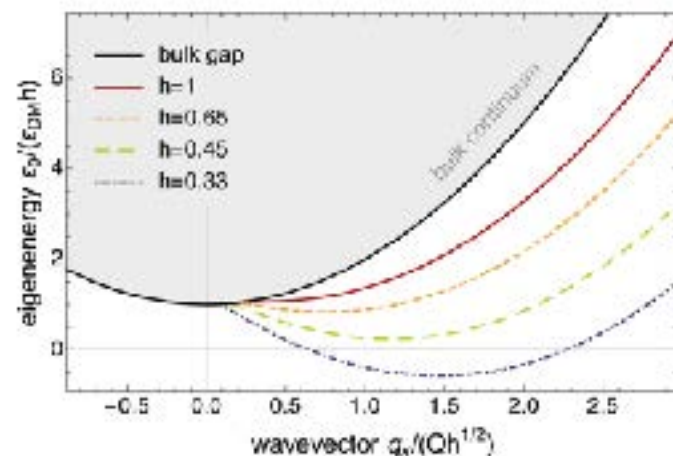
$$\hat{n} \vec{\nabla} \hat{M} - Q \hat{n} \times \hat{M} = 0$$

Rohart & Thiaville, PRB (2013)
Meynell et al PRB (2014)



twist of the magnetisation
even in the field-polarised state

attractive potential for spin waves:

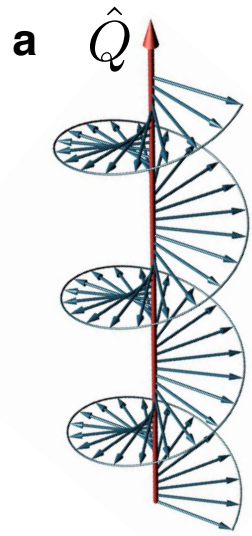


creation of skyrmions using
condensation of edge magnon

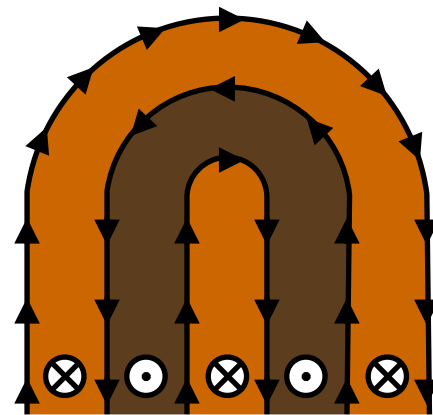
magnon localised to the edge

Müller, Rosch, MG, New J. Phys. (2016)

Topological defects of helimagnetic order

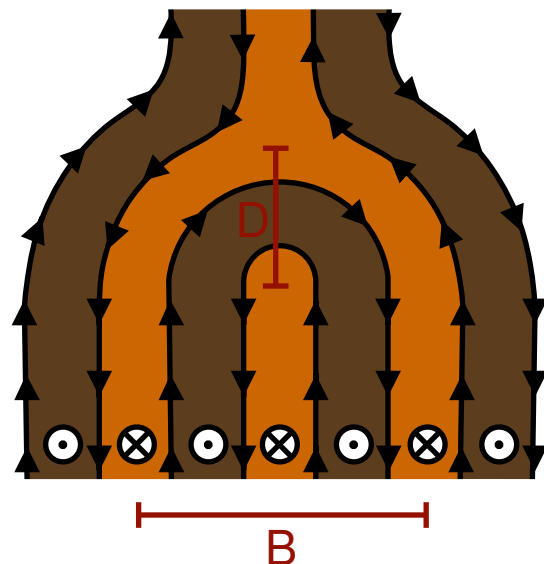


magnetic helix = **lamellar structure** similar to cholesteric liquid crystals or stripe domain patterns



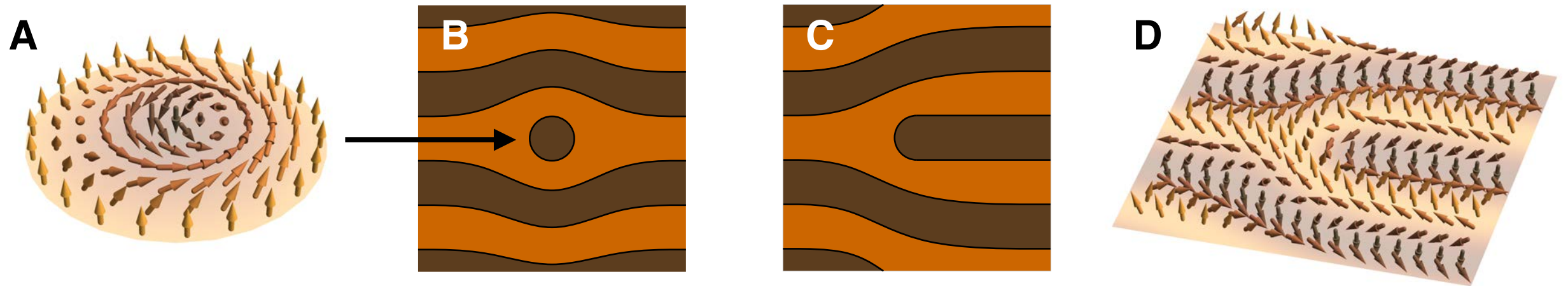
pitch \hat{Q} is a director

$\Rightarrow \pm\pi$ vortices are possible
= **disclinations defects**



disclinations combine to form a **dislocation** with Burgers vector B

Skyrmion winding number of dislocations



skyrmion number $W = -1$

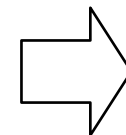
skyrmion embedded in a topologically trivial background

skyrmion number $W = -1/2$

dislocation (with $B = \lambda$) = meron

general relation for skyrmion number of dislocation with Burgers vector B :

$$|W| = \frac{1}{2} \text{mod}_2 \left(\frac{B}{\lambda} \right)$$

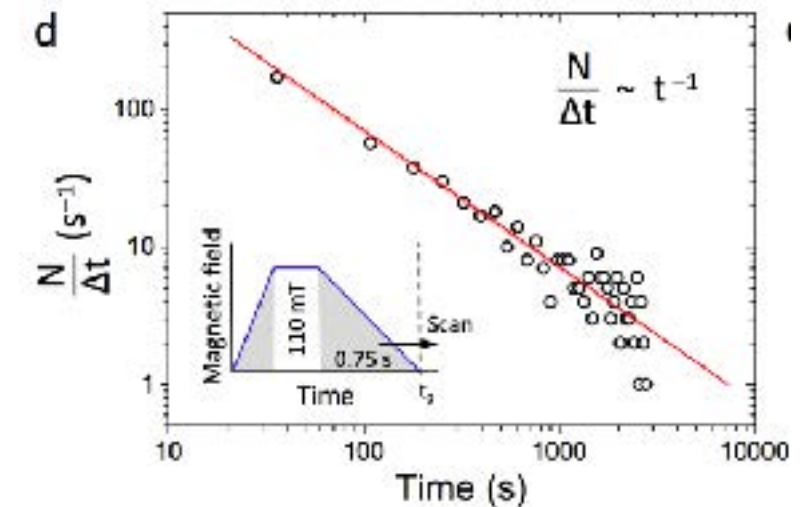
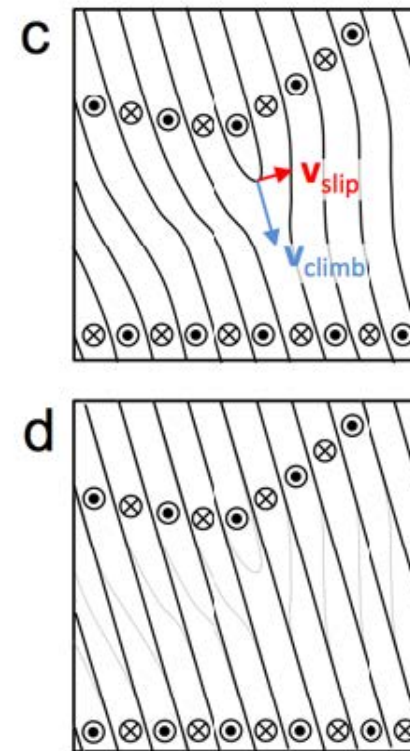
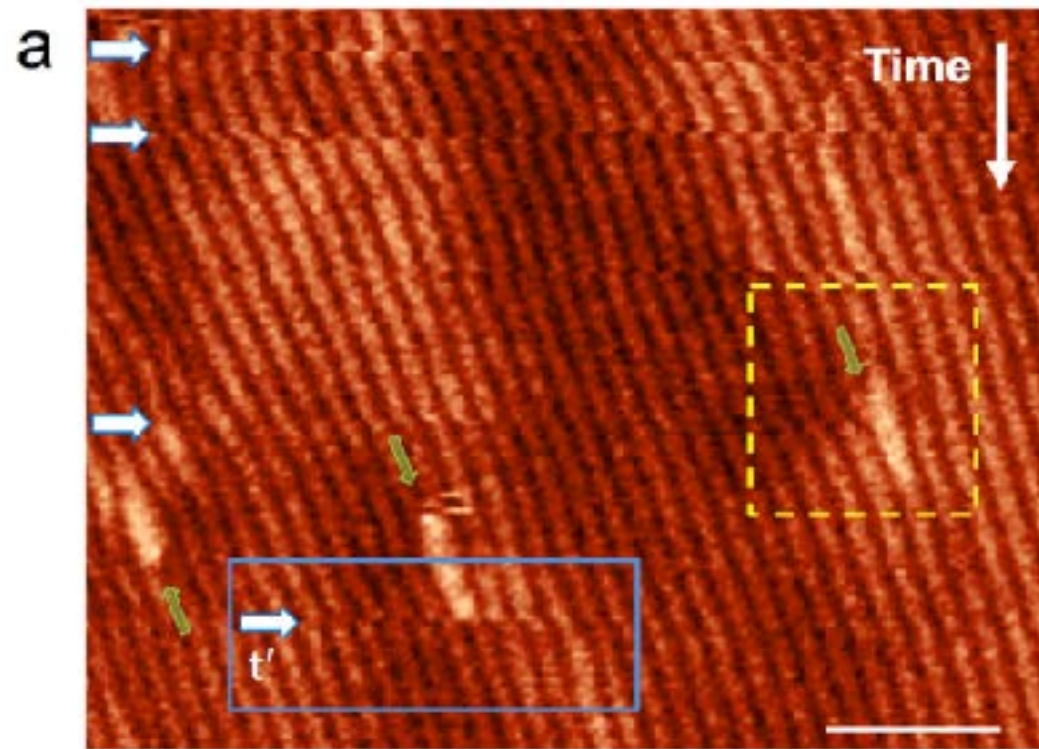


only dislocations with half-integer B contribute to

topological Hall effect & emergent electrodynamics

Magnetic relaxation by climb motion of dislocations

magnetic force microscopy: surface of FeGe



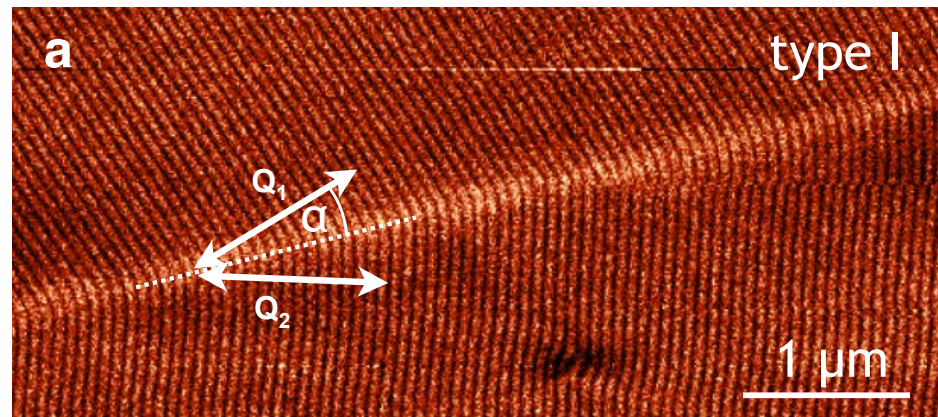
slow power-law relaxation
~ 1000 sec!

climb motion of dislocations \Rightarrow 180° phase shift

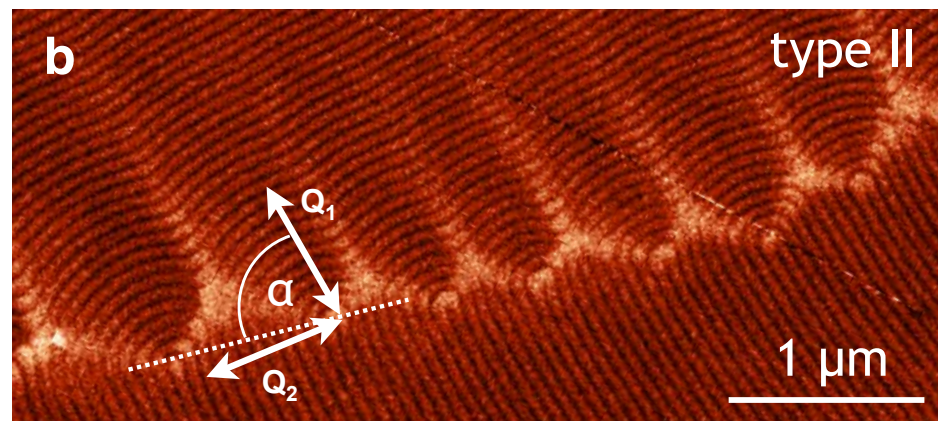
A. Dussaux et al., Nat Comm 2016

Topological domain walls

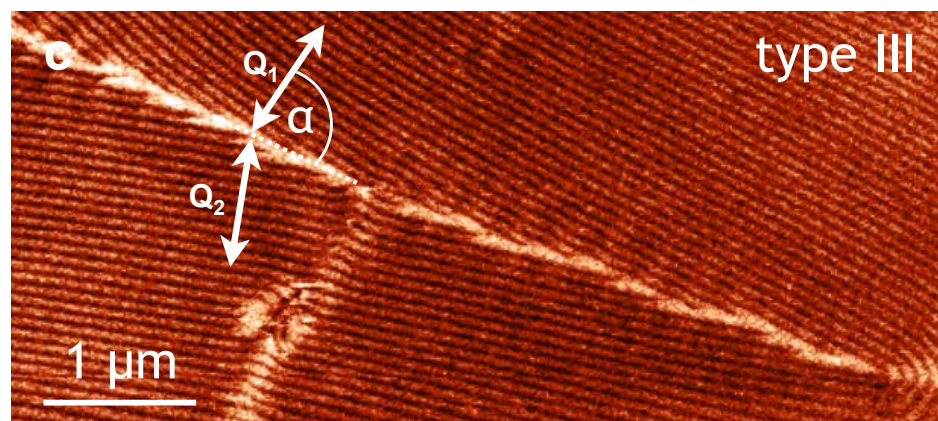
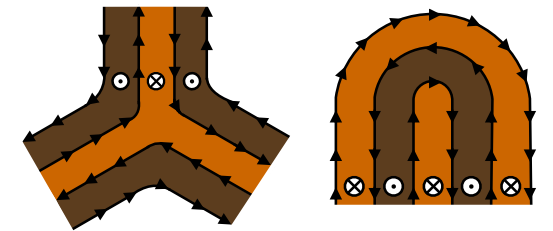
depending on relative angle: three types of domain walls



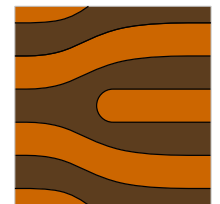
curvature wall



zig-zag
disclination wall



dislocation wall

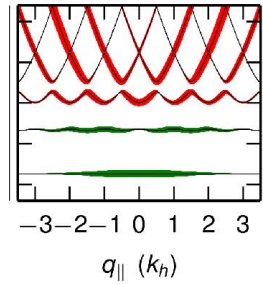


MFM: surface of FeGe

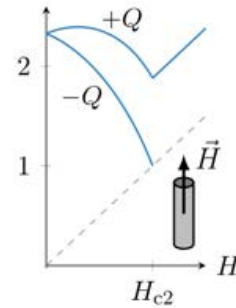
P. Schoenherr, ... MG, D. Meier, [arXiv:1704.06288](https://arxiv.org/abs/1704.06288)

Summary:

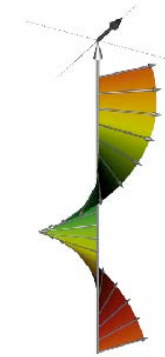
- Spin-wave dynamics of the magnetic helix



band structure

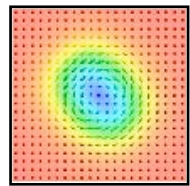


resonances

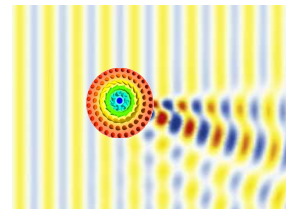


linear polarisation

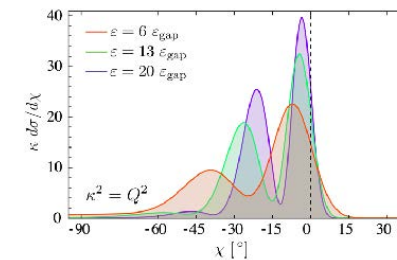
- Spin-wave dynamics of the magnetic skyrmion



internal modes

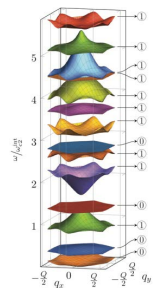


skew scattering



rainbow scattering

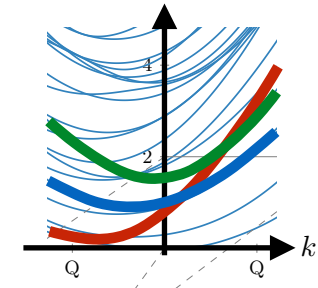
- Spin-wave dynamics of the magnetic skyrmion lattice



topological band structure



resonances



non-reciprocity