Electron correlations in low-dimensional topological systems

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SPIN, CHARGE, AND ENERGY CURRENTS IN NOVEL MATERIALS,

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Collaborators

main

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Supported by Topological Materials Science
トポロジーが紡ぐ物質科学のフロンティア

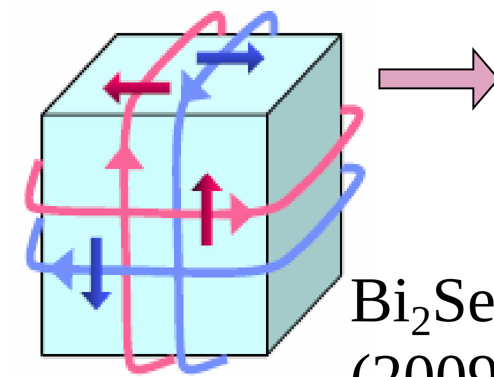
Topological materials and exotic edge states

Topological Insulators

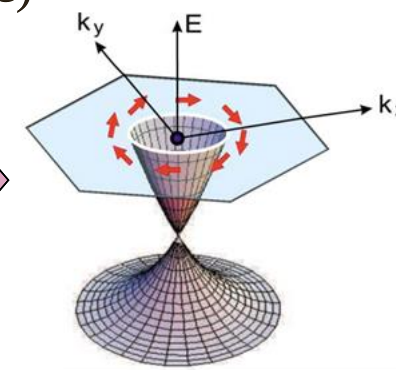
2D (Kane-Mele, Bernivig et al), 3D (Fu-Kane-Mele, etc)

Bulk insulator

Topologically protected surface state



Bi_2Se_3
(2009)



surface

Exotic edge (surface) states	
QHE (2D):	**Chiral quasi particles**
Insulators (3D):	**Dirac quasi-particles**
Superconductors:	**Majorana quasi-particles**

Topological Insulators: weakly correlated

■ Topologically nontrivial properties

Topological numbers

1D:	polyacetylene	winding no.
2D:	HgTe/CdTe, GaSb/InAs	Z_2
3D:	$Bi_{1-x}Sb_x$, Bi_2Se_3, Bi_2Te_3, etc	Z_2

Topologically protected edge states

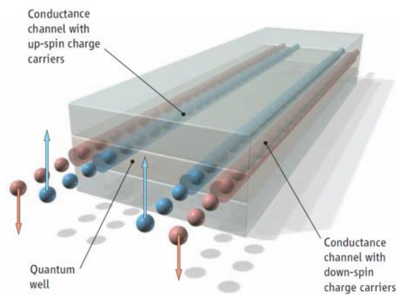
↓

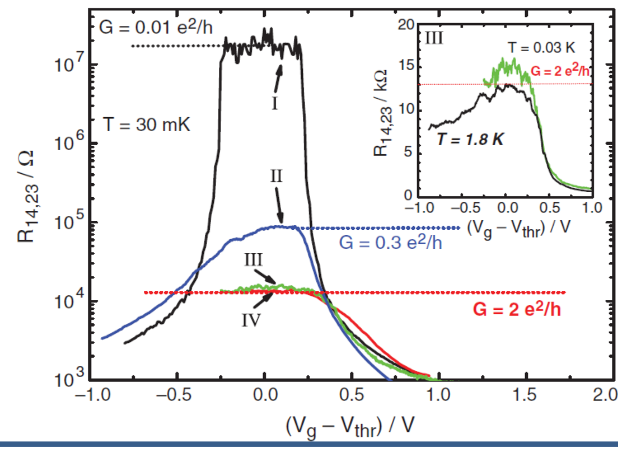
Spintronics, Quantum information, etc

Observation of edge states in TIs

HgTe/CdTe quantum well (2D)

M.Kronig et al., Science 318(2008)766



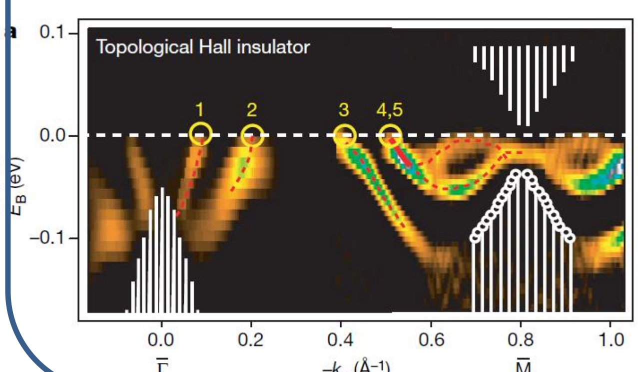


Conductance carried by edge states

Spin-orbit interaction is essentially important

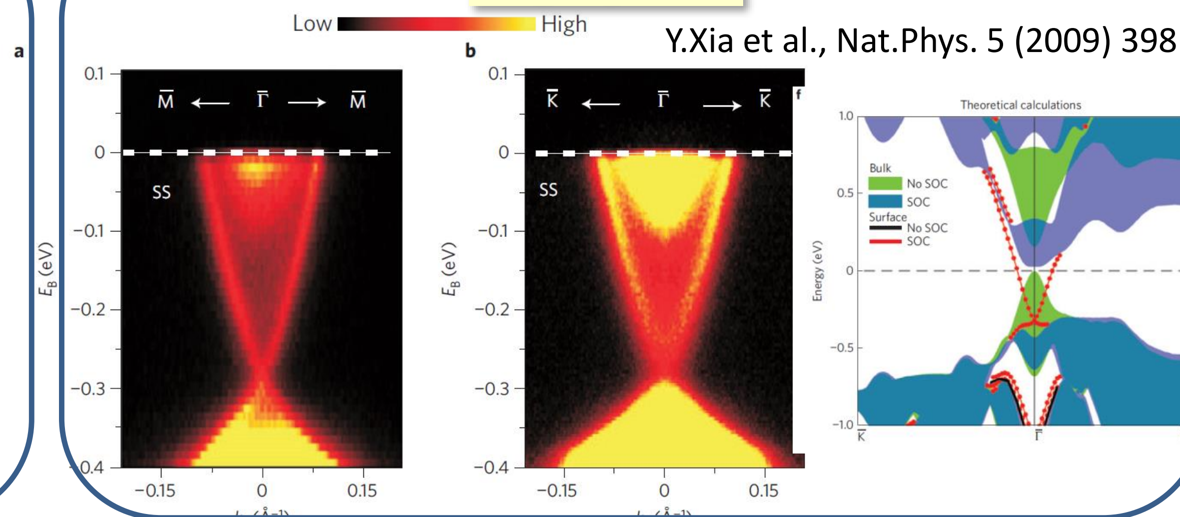
$Bi_{1-x}Sb_x$ (3D)

D.Hsieh et al., Nature 452 (2008) 970



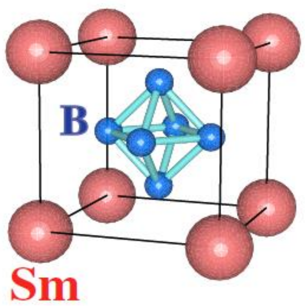
Bi_2Se_3 (3D)

Y.Xia et al., Nat.Phys. 5 (2009) 398



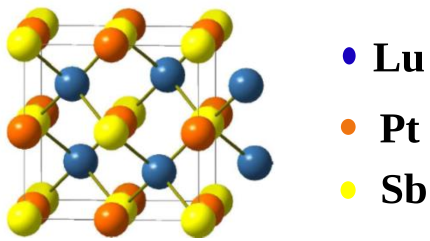
Topological phases in correlated electron systems

SmB_6 (Kondo insulator)



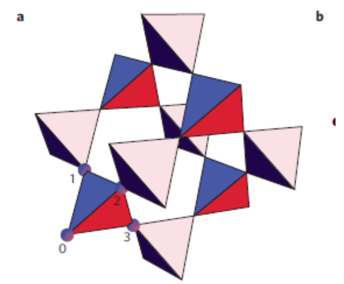
Dzero *et al.* PRL (2010)

LuPtBi (Heusler compound)



Chadov et al. Nature Materials (2010)
Lin et al. Nat. Mat. (2010)

$A_2Ir_2O_7$ (A=Pr, Eu)



Pesin and Balents
Nature Phys. (2010)

Electron correlations + SO coupling

How the interaction modifies the nature of topological insulators (superconductors) ?

1. Same topological phase, different physics

   e.g. Topological Mott insulators

2. Reduction of topological classification

   e.g. 1D Kitaev chain $Z \Rightarrow Z_8$

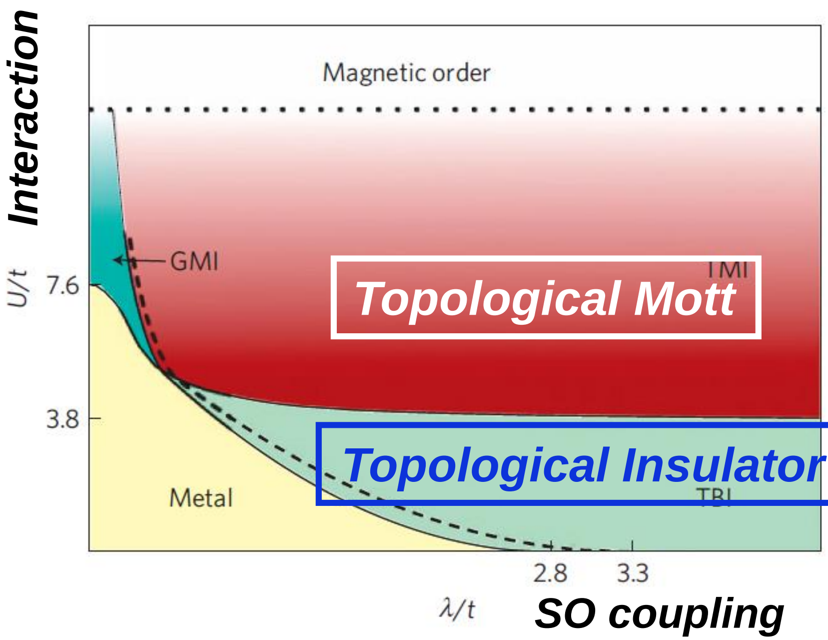
etc.

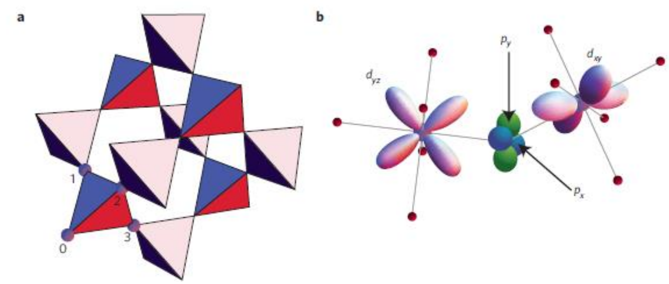
① Topological Mott insulators

① Topological Mott insulator

D. Pesin and L. Balents, Natute Phys. (2010)

Mott physics and band topology in materials with strong spin–orbit interaction





Ir-based pyrochlores

$$H = \sum_{Ri\alpha}(\varepsilon_\alpha - \mu) d^\dagger_{Ri\alpha} d_{Ri\alpha} + t \sum_{\substack{\langle Ri,R'i'\rangle \\ \alpha\alpha'}} T^{ii'}_{\alpha\alpha'} d^\dagger_{Ri\alpha} d_{R'i'\alpha'} + \frac{U}{2}\sum_{Ri}\left(\sum_\alpha d^\dagger_{Ri\alpha} d_{Ri\alpha} - n_d\right)^2$$

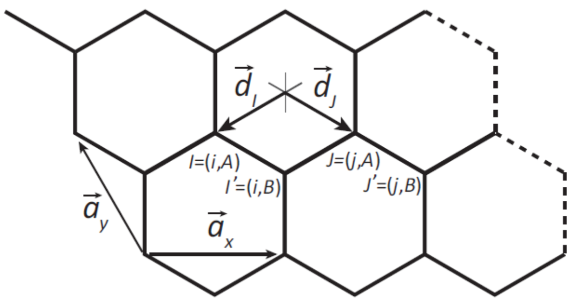
Slave rotor mean field

Bulk state: **gapful**
Edge state: **gapless spin excitations**

→ **Edge Mott state**

① Topological Mott insulator

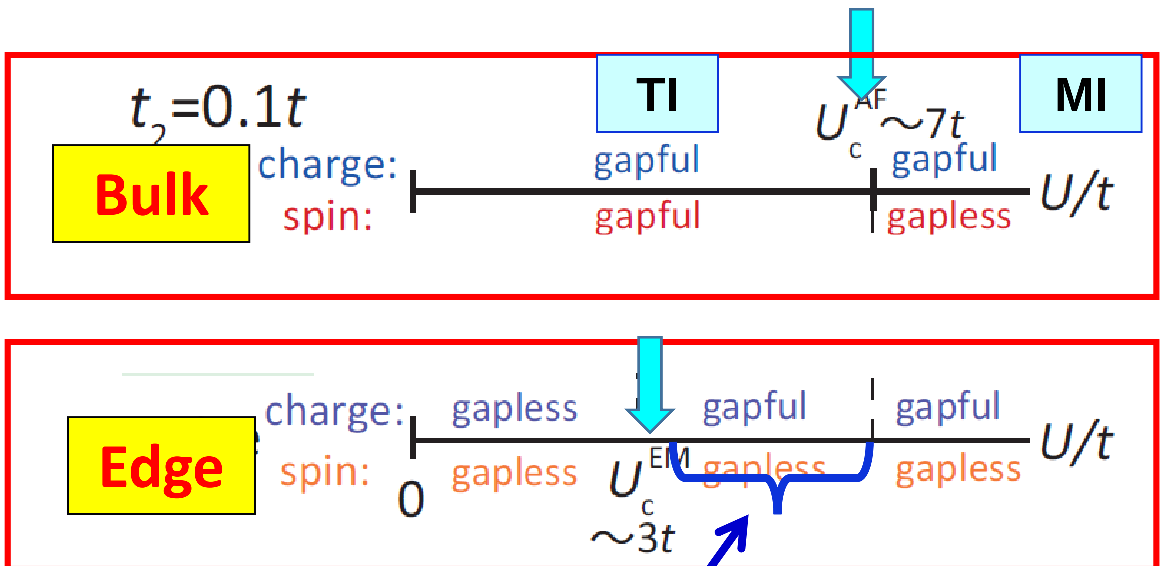
Honeycomb lattice



Kane-Mele-Hubbard (2D)

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\rm KM} + U\sum_I \hat{n}_{I\uparrow}\hat{n}_{I\downarrow},$$

$$\hat{\mathcal{H}}_{\rm KM} = -t\sum_{\langle I,J\rangle\sigma} \hat{c}^\dagger_{I\sigma}\hat{c}_{J\sigma} + it_2 \sum_{\langle\langle I,J\rangle\rangle\alpha\beta} \nu_{ij}\hat{c}^\dagger_{I\alpha}[\sigma_z]_{\alpha\beta}\hat{c}_{J\beta}$$



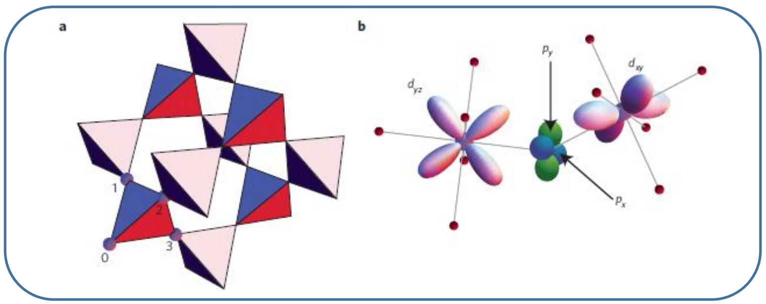
Edge Mott insulator **is proposed**

(bulk = TI, edge = Mott)

Y. Yamaji et al. PRB(R) 2011

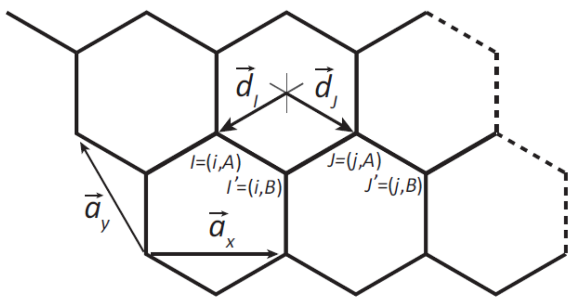
Topological Mott insulators
have not been confirmed yet
even theoretically

Ir-based pyrochlores



may be candidates for
Weyl semimetals

2D simple models with interaction



Kane-Mele model

Bernevig-Hughes-Zhang model

may not be topo Mott insulators

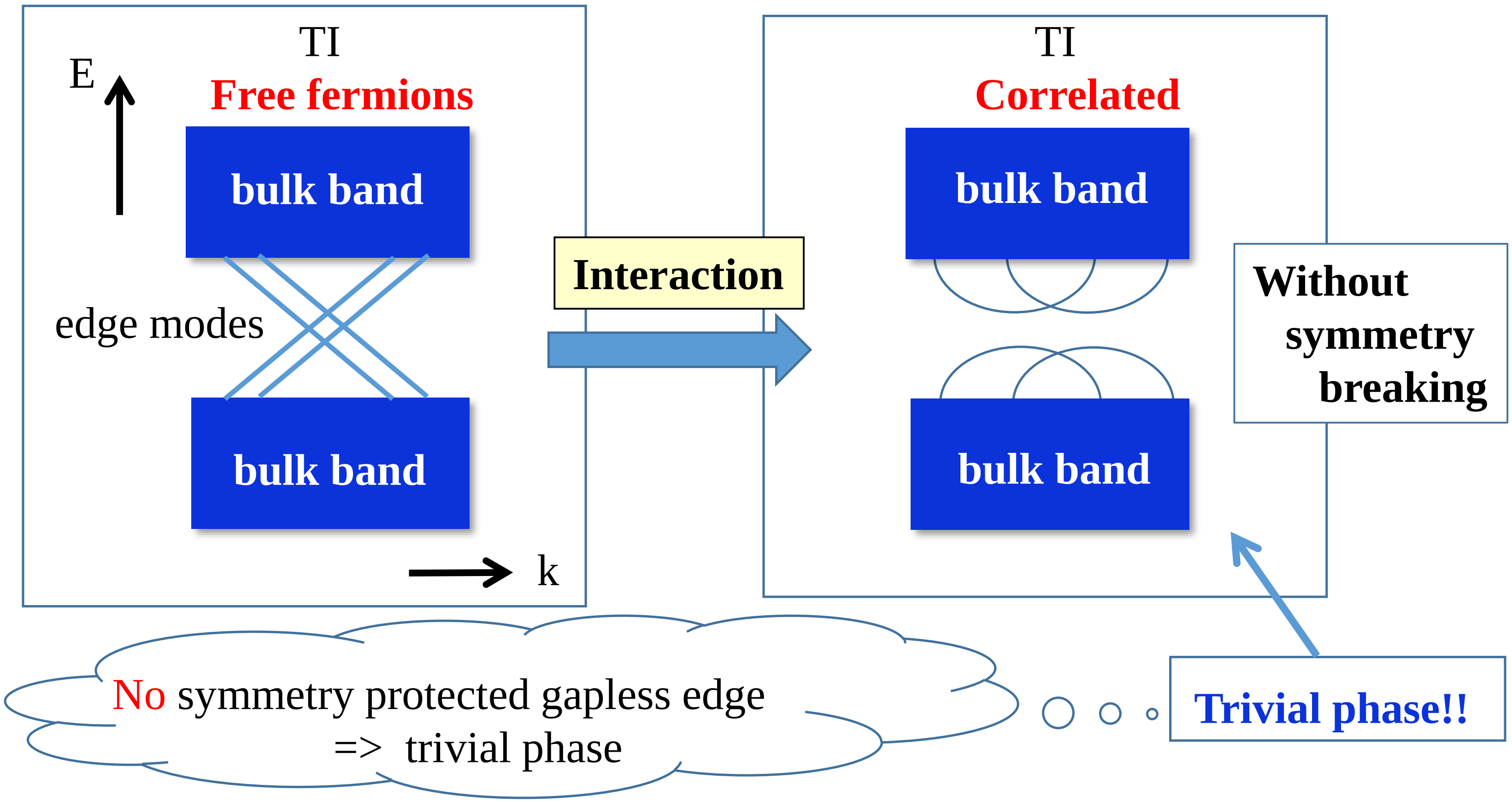
cf Yamaji et al. 2011

② Reduction of Classification

② Reduction of classification

Fidkowski and Kitaev (2010)

For finite interaction, whether a system is topological (trivial):
by examining presence (absence) of symmetry protected edge modes



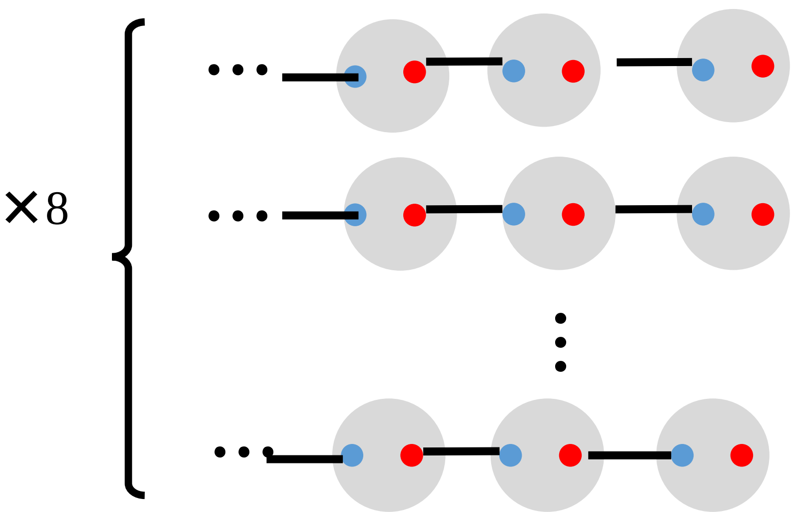
Reduction of classification

e.g., 1D class BDI (**Kitaev Chain**) $\mathbb{Z} \rightarrow \mathbb{Z}_8$

② Reduction of classification

Kitaev chain (TRS, PHS)

Fidkowski and Kitaev (2010)



Majorana modes

classification Z
=[# of gapless edges]

	# of gapless edges							Classification
Free-fermions	1	2	···	8	9	10	···	Z
Correlated fermions	1	2	···	0	1	2	···	Z_8

Kitaev chain ×8 :
topologically trivial!

$\mathbb{Z} \rightarrow \mathbb{Z}_8$

1D, 2D, 3D

No experimental test bed !

Contents

Correlation Effects in Topological Insulators/Superconductors

1. Topological Mott Insulator

- Topological Mott insulator
- Edge Mott states 1D & 2D

- T-induced change in Fermi-Bose statistics 2D

2. Reduction of Topological Classification

$CeCoIn_5/YbCoIn_5$ superlattice
a testbed for reduction of topological classification

Topological Mott Insulator in one dimension

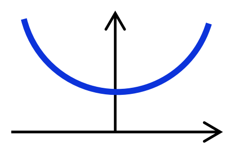
Chiral symmetry protected TI

Yoshida, Peters, Fujimoto, NK

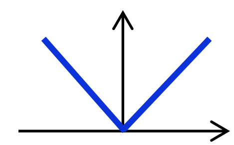
Key words

Topological insulator (noninteracting)

Bulk : gapful



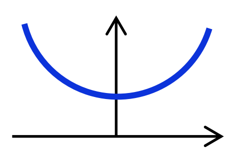
Edge : gapless



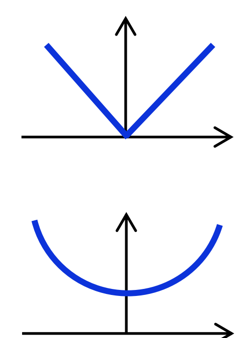
Topological Mott insulator (interacting)

Edge Mott state

Bulk : gapful



Edge : spin gapless
charge gapful



1D topological insulator

e.g. *chiral symmetry protected* BDI

Su-Schrieffer-Heeger Model

Correlation ?

1D correlated electron systems

- topological Mott insulator
- edge Mott state
- interaction-driven topological transition

Topological properties

Topological invariant defined by Green's function

Entanglement spectrum

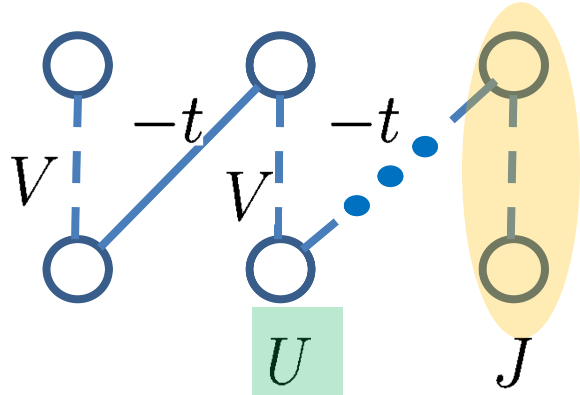
cf. S.Manmana *et al.*
PRB 2012
Time-dep. DMRG

Correlated Su-Schrieffer-Heeger Model

(chiral symmetric)

$$H = H_{SSH} + U\sum_{i\alpha} n_{i\alpha\uparrow}n_{i\alpha\downarrow} + J\sum_{i} \boldsymbol{S}_{ia}\cdot\boldsymbol{S}_{ib}$$

$$H_{SSH} = \sum_{i\sigma}(-tc^{\dagger}_{i+1a\sigma}c_{ib\sigma} + Vc^{\dagger}_{ia\sigma}c_{ib\sigma} + h.c.)$$



Noninteracting case *U=J=0*

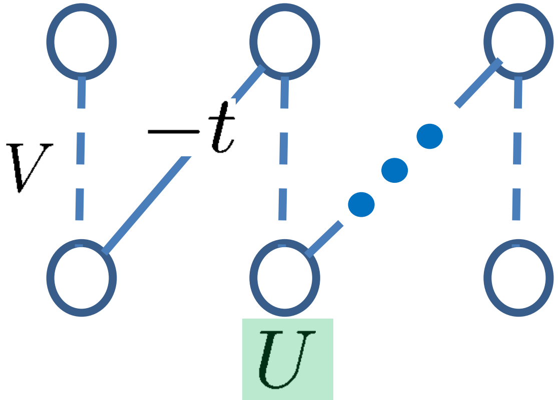
Topological phase $-t < V < t$

S.Manmana *et al.*
PRB 2012

Correlation effects : DMRG

Powerful tool for calculation of
ground states, correlation function etc..

Hubbard interaction

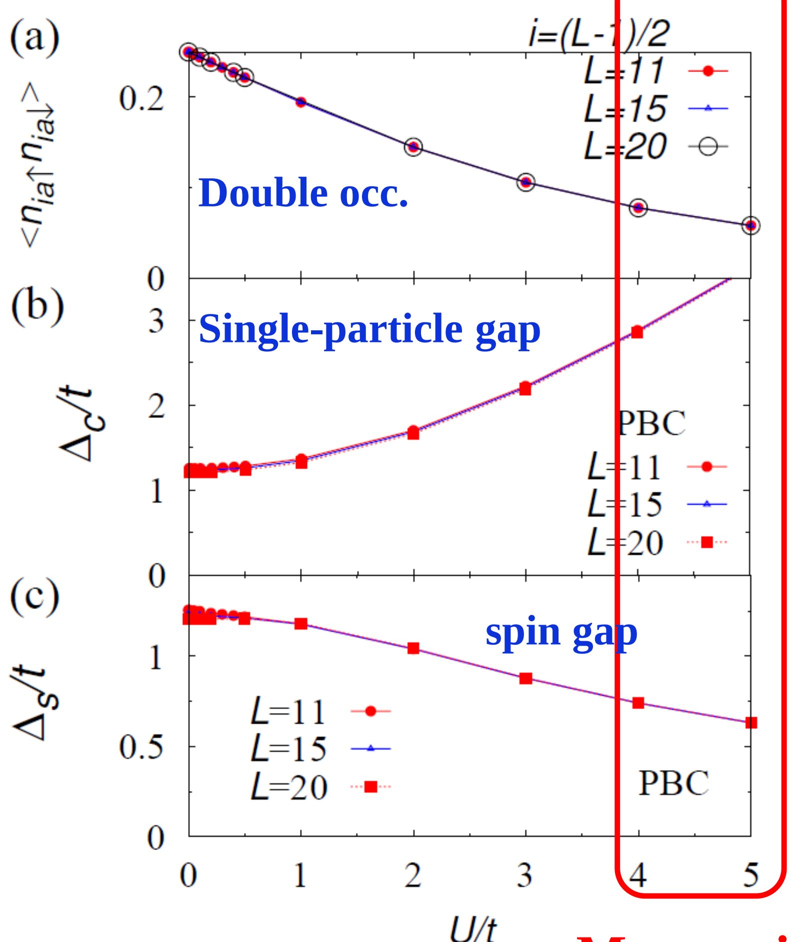


- topological Mott insulator
- edge Mott state

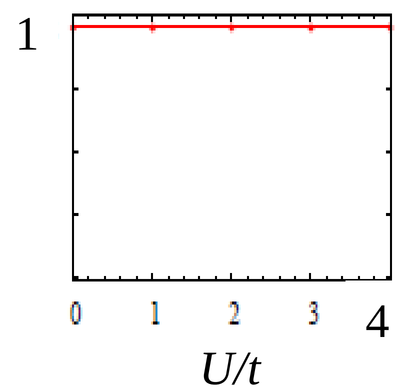
U-dependence (J=0)

For *V=-0.4/t*, nontrivial at *U=0.*

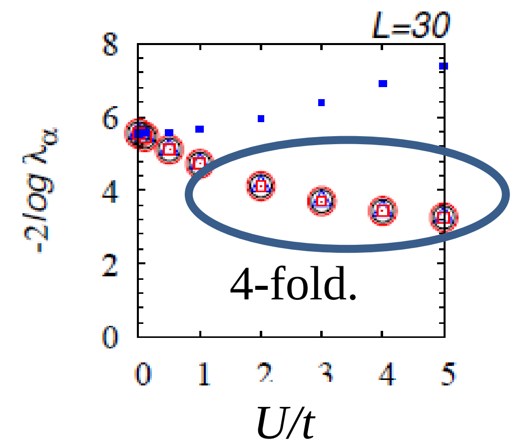
Bulk Properties



Winding #



Entanglement



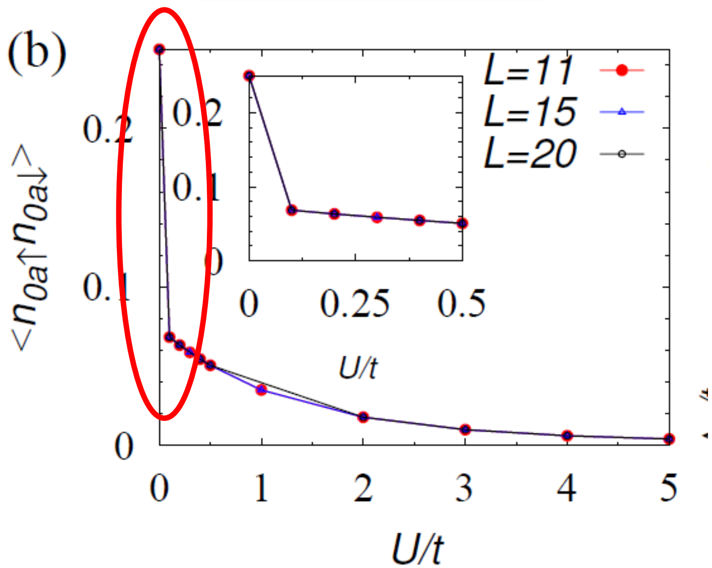
Nontrivial phase

Crossover to

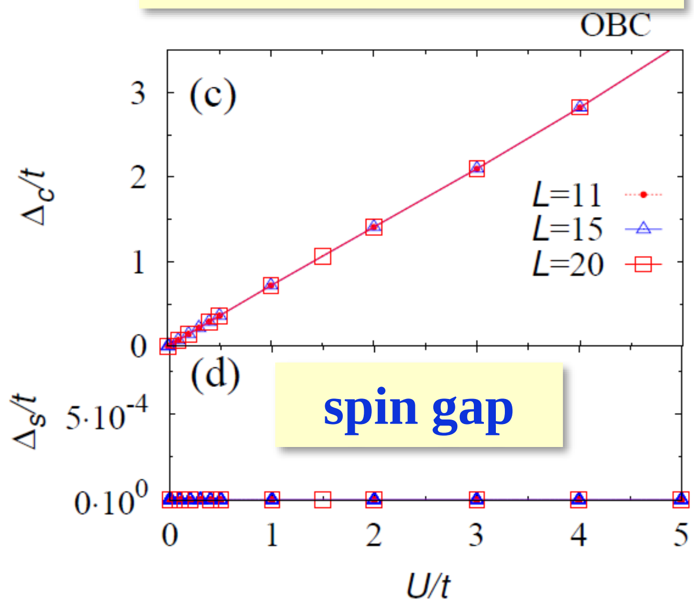
Topological Mott ins.

Edge Properties

Double occ.



Single-particle gap



Charge gapful

Spin gapless

Topological Mott insulator

Edge Mott state emerges!

still correlated

1D topological insulator

chiral symmetry BDI

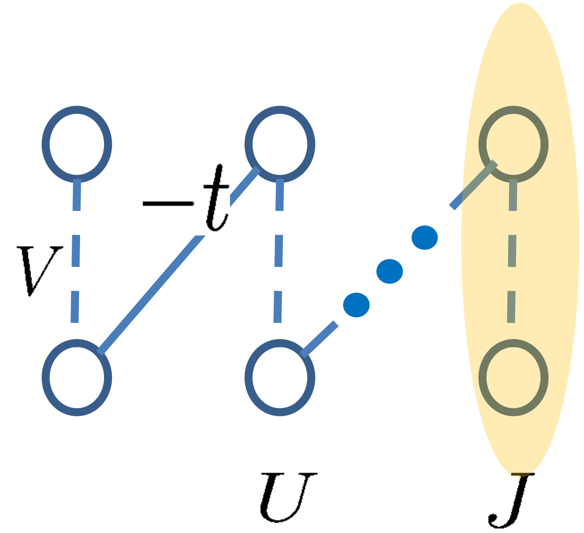
Correlated Su-Schrieffer-Heeger Model

a solid example
Topological Mott
with **edge-Mott state**

	Noninteracting	Interacting
	Chiral-symmetry d TI	Chiral-symmetry protected TI
Edge	Chi l-symmetry Shiba-symmetry protected **Edge state**	Chiral-symmetry ~~Shiba-symmetry~~ protected **Edge Mott state**

Spin ↔ **charge**
U>0 **U<0**

Hubbard interaction
+ Exchange interaction

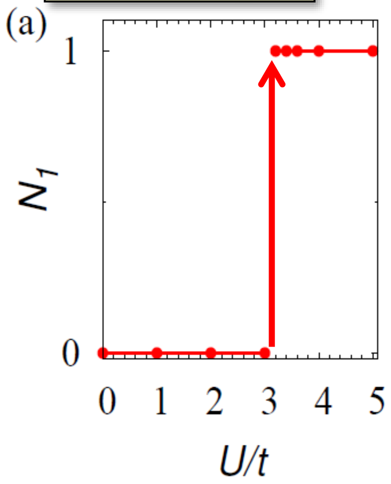


- topological phase transition

For *V=-1.6t* ;
trivial at U=0, J=0.

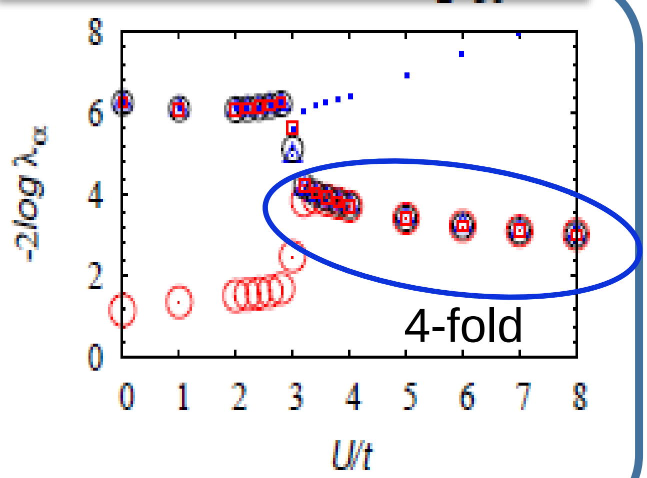
Numerical results for U>0, J<0 (ferromagnetic)

Winding #



Top phase transition

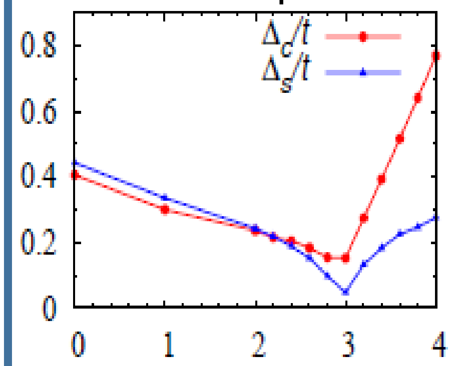
Entanglement spectrum



Edge-Mott state

Gap

Single particle:
Spin:



No gap closing in DOS

Interaction-induced topological Mott transition
without Gap closing in DOS

a solid example
Topological Mott transition

Gap closing of spinon excitations
Zeros of *G(iω,k)* changes topo. properties.

cf Haldane-gap phase (large U limit)

Summary of 1D systems

Topological Mott insulator
1D chiral symmetric class

- winding #
- entanglement spectrum

- Emergence of edge Mott states

◇ charge gap
◇ spin gapless

Absence of "Shiba symmetry"

- Unconventional topological phase transition

◇ no gap-closing in DOS
◇ gap-closing of spinons

Topological Mott phase in 2D

~ bilayer system~

T. Yoshida and NK (2016)

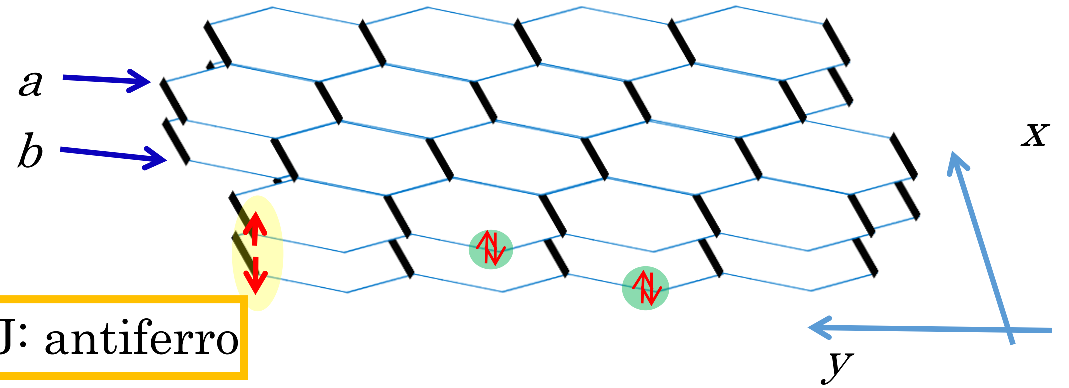
H-Q. Wu, Z-Y. Meng, T. Yoshida, NK et al. (2016)

cf (double-layer graphen), Z. Bi et al. (2016)

Model (Bilayer Kane-Mele model with interaction)

spin-Hall ins. ×2

DMFT +CTQMC



J: antiferro

$$H = H_0 + H_{\rm int},$$

$$H_0 = -\sum_{\langle i,j\rangle,\alpha} t_{i,j} c^\dagger_{i,\alpha,\sigma} c_{j,\beta,\sigma} + it_{so} \sum_{\langle\langle i,j\rangle\rangle} \boldsymbol{\sigma}_{\sigma\sigma'} \cdot \hat{\boldsymbol{d}}_i \times \hat{\boldsymbol{d}}_j / |\hat{\boldsymbol{d}}_i \times \hat{\boldsymbol{d}}_j| c^\dagger_{i,\alpha,\sigma} c_{j,\beta,\sigma'},$$

For x-direction $t_{i,i+e_x} = t$

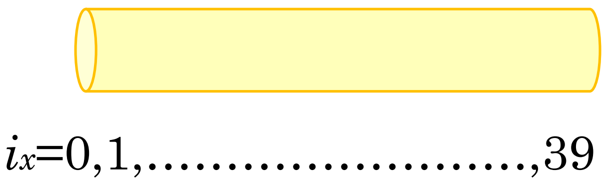
For other direction $t_{i,j} = 0.7t$

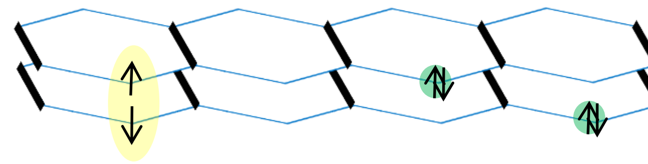
Bulk properties

K. Slagle *et al.*, PRB (2015)
Y.-Y. He, *et al.*, PRB (2016)

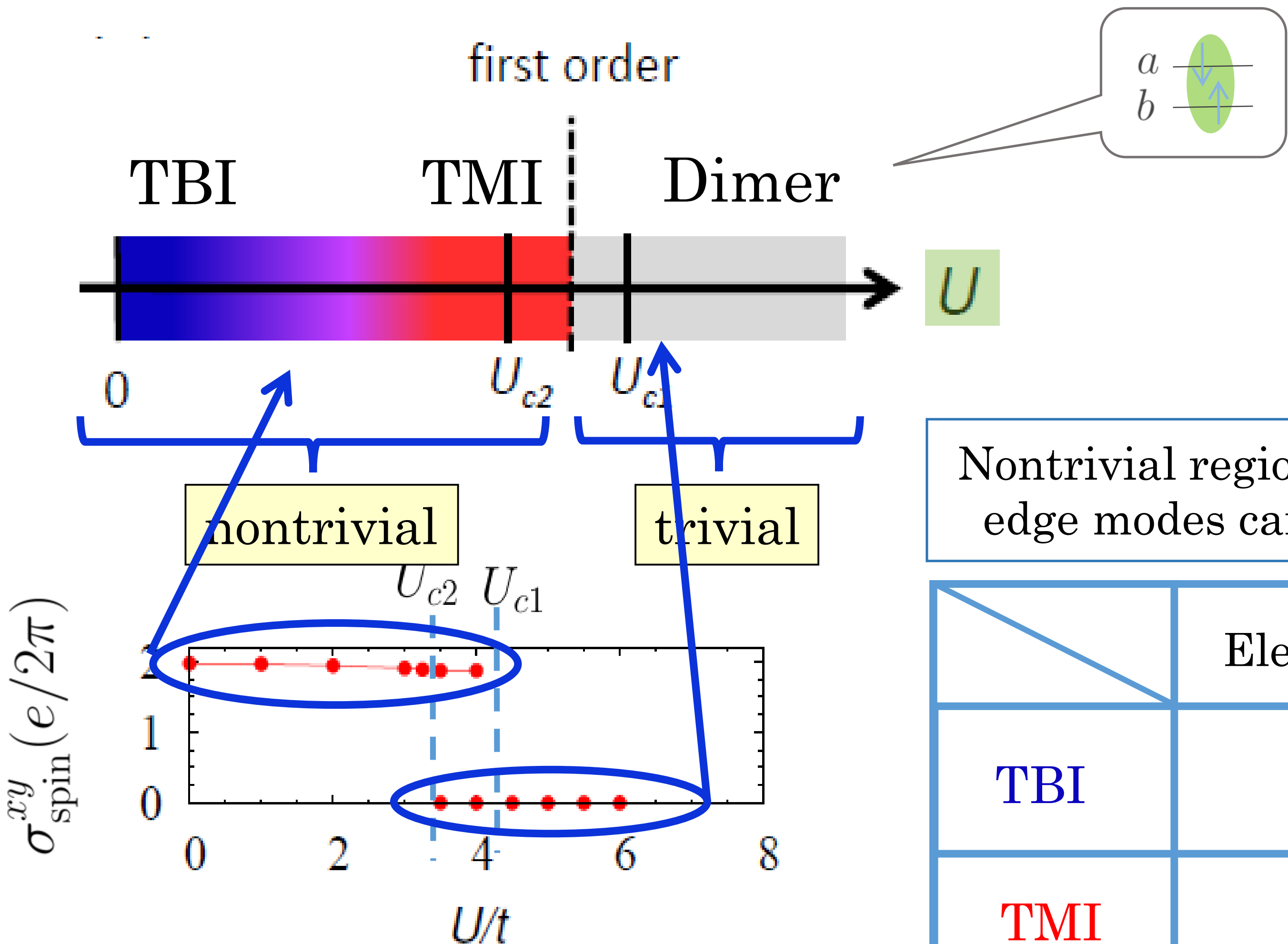
Phase diagram at *T=0.05t*

$J = t \quad t_{so} = 0.2t$





DMFT
+CTQMC



Nontrivial region:
edge modes carry spin current.

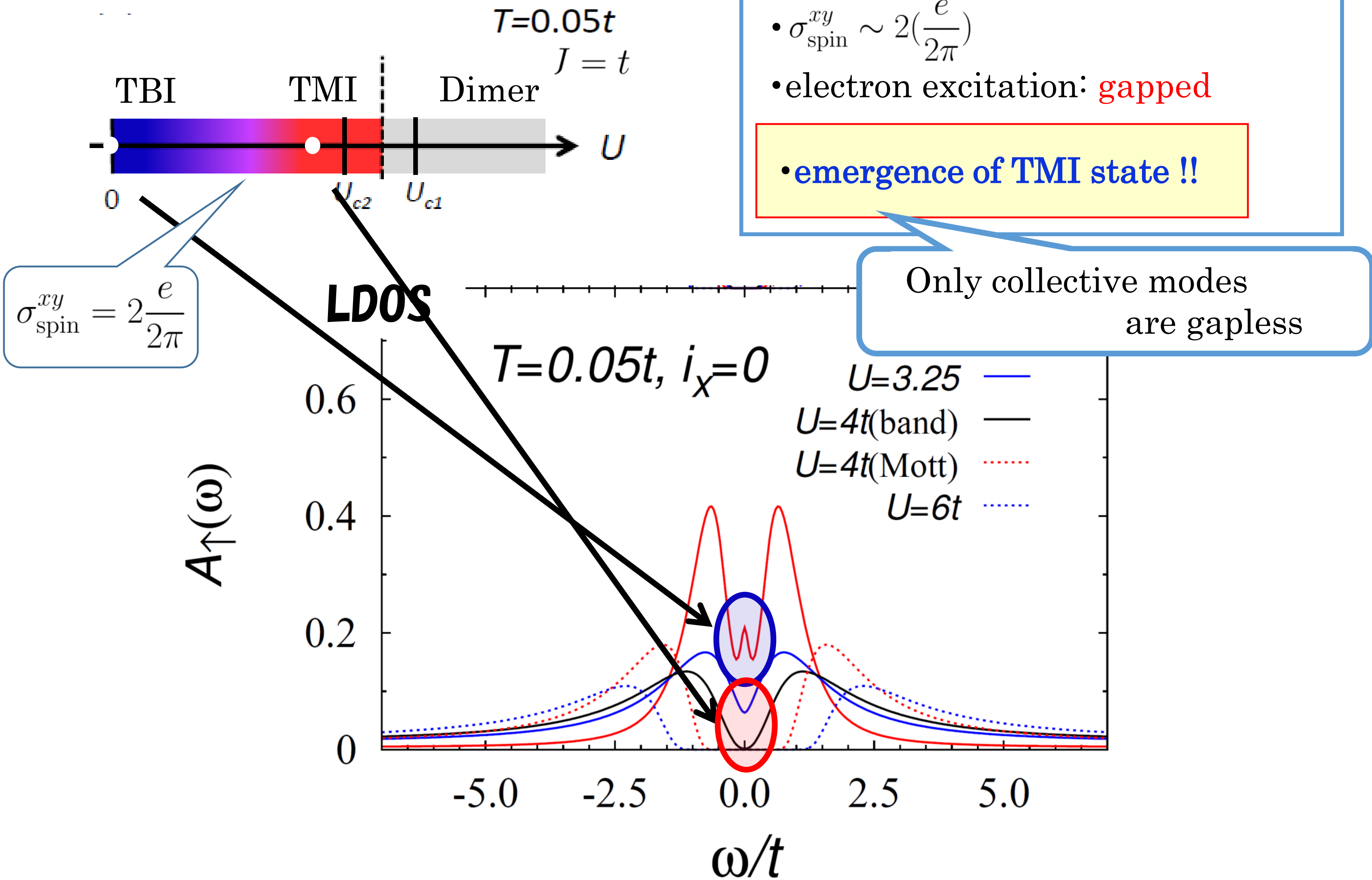
	Electron edge modes
TBI	gapless
TMI	gapped

Electron excitations @ edge

$J = t \quad t_{so} = 0.2t$

- $\sigma^{xy}_{\rm spin} \sim 2(\frac{e}{2\pi})$
- electron excitation: gapped
- **emergence of TMI state !!**

Only collective modes are gapless

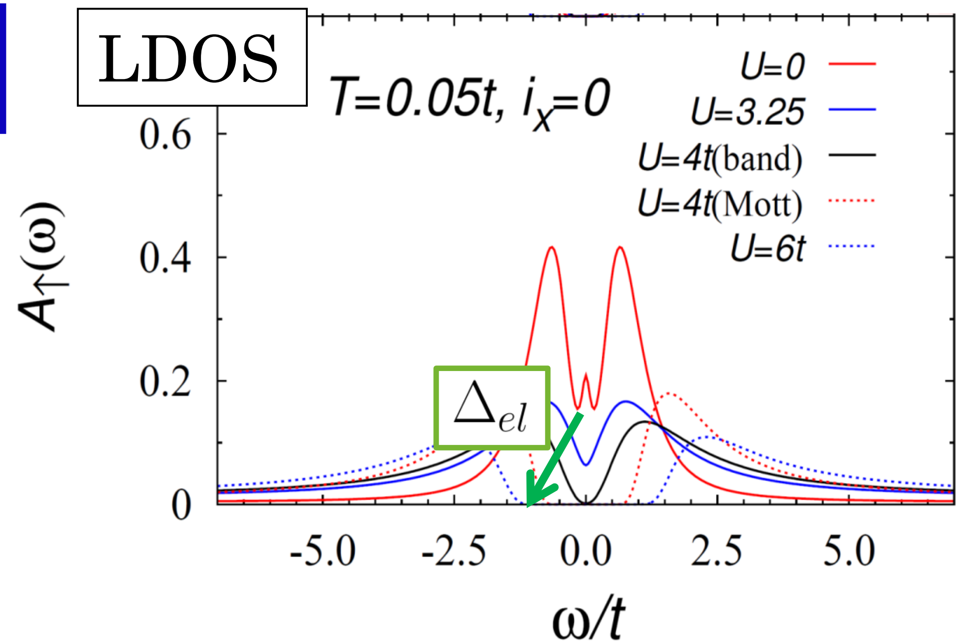


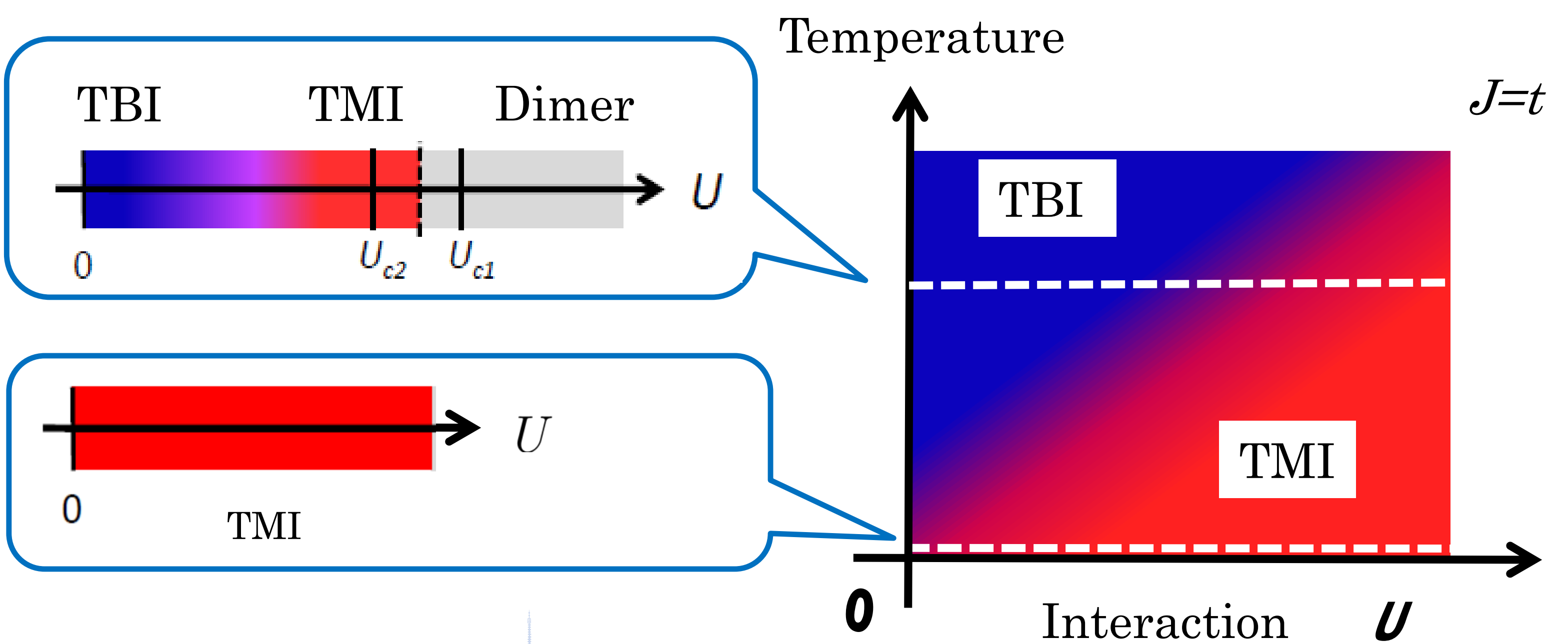
Temperature effects

Δ_{el} :increases

⇕

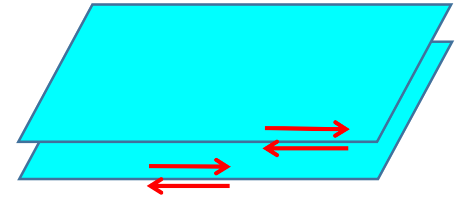
decrease of $T_{eff} = T/\Delta_{el}$



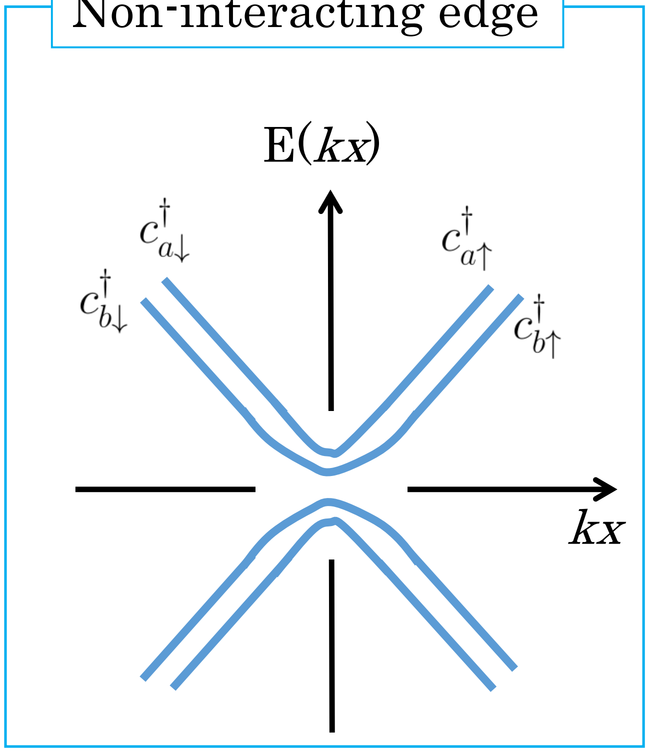


T=0: bosonization analysis

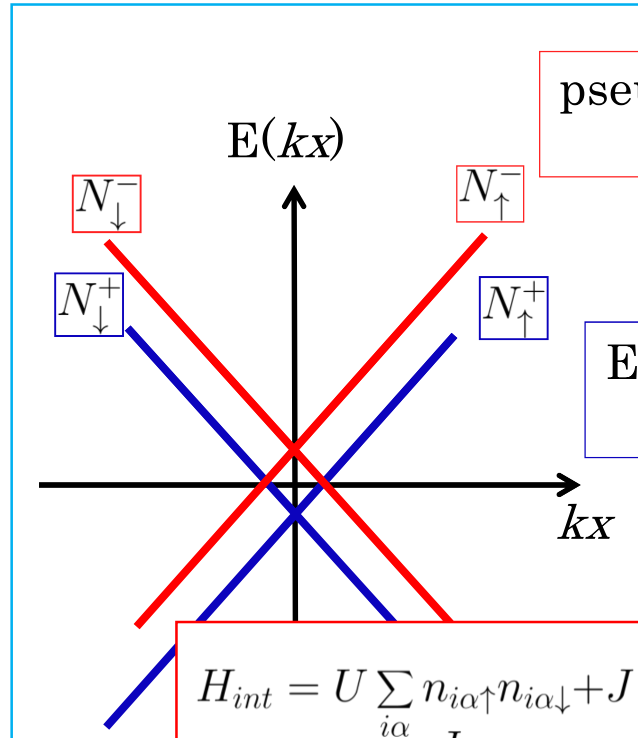
$N_\sigma^{\pm} = n_{a\sigma} \pm n_{b\sigma}$



Non-interacting edge



bosonization



pseudo-spin density

Electron density

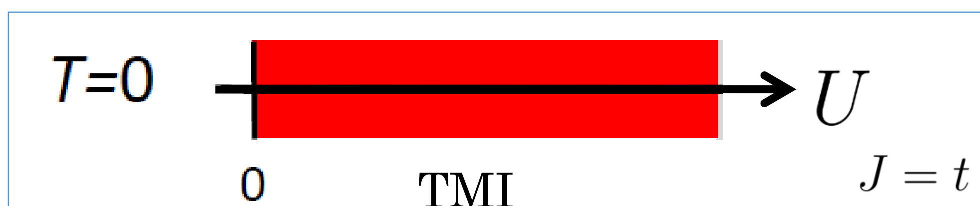
$$H_{int} = U\sum_{i\alpha} n_{i\alpha\uparrow}n_{i\alpha\downarrow} + J\sum_i S_{ia}^z S_{ib}^z + \frac{J}{2}\sum_i (S_{ia}^+ S_{ib}^- + S_{ia}^- S_{ib}^+)$$

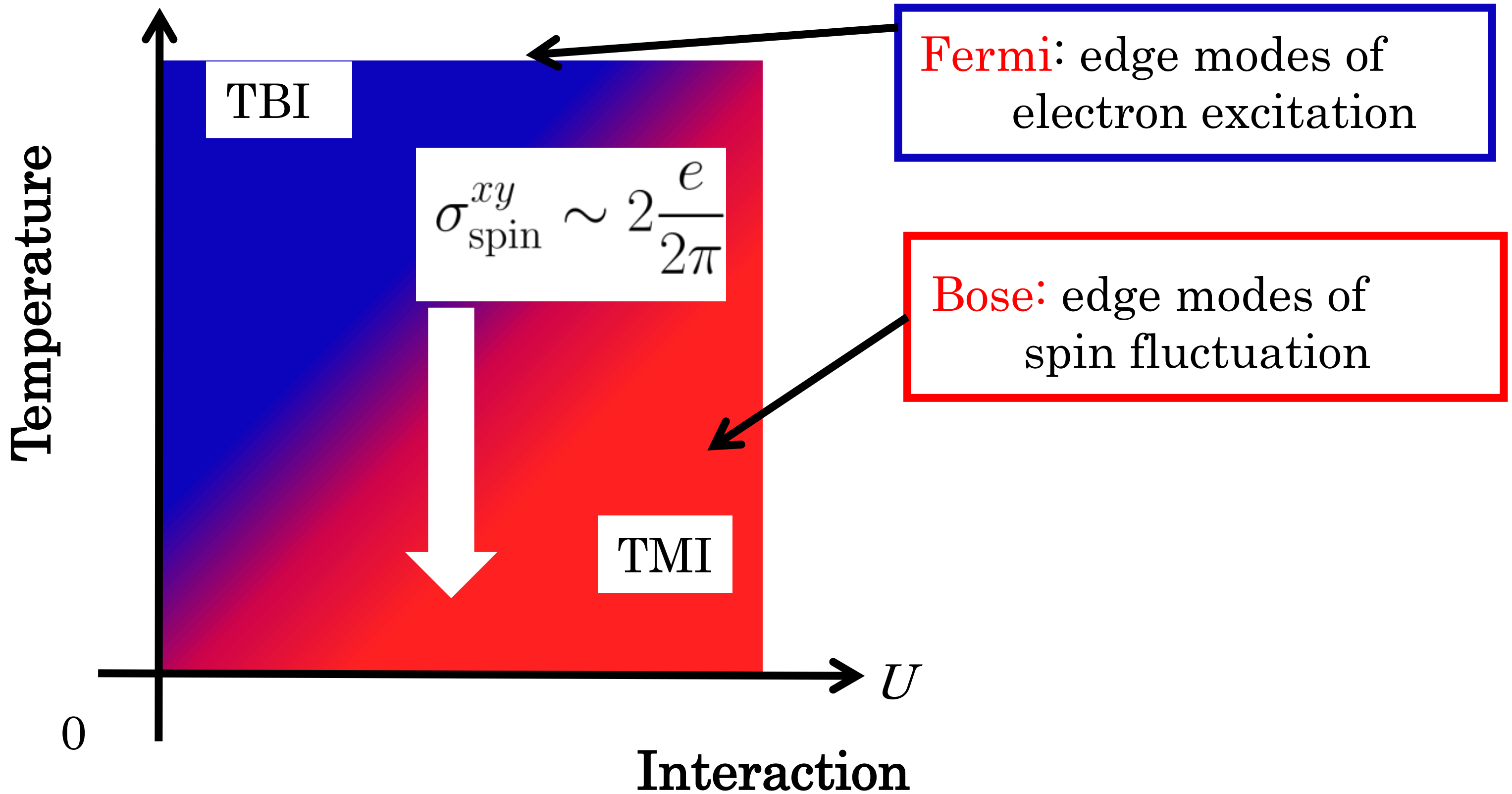
Density excitations (collective excitation) are gapless

$\langle T N_\sigma^+(\tau) N_\sigma^+ \rangle \quad \sigma = \uparrow, \downarrow$

Similar analysis

Z. Bi *et al.*, arXiv (2016)





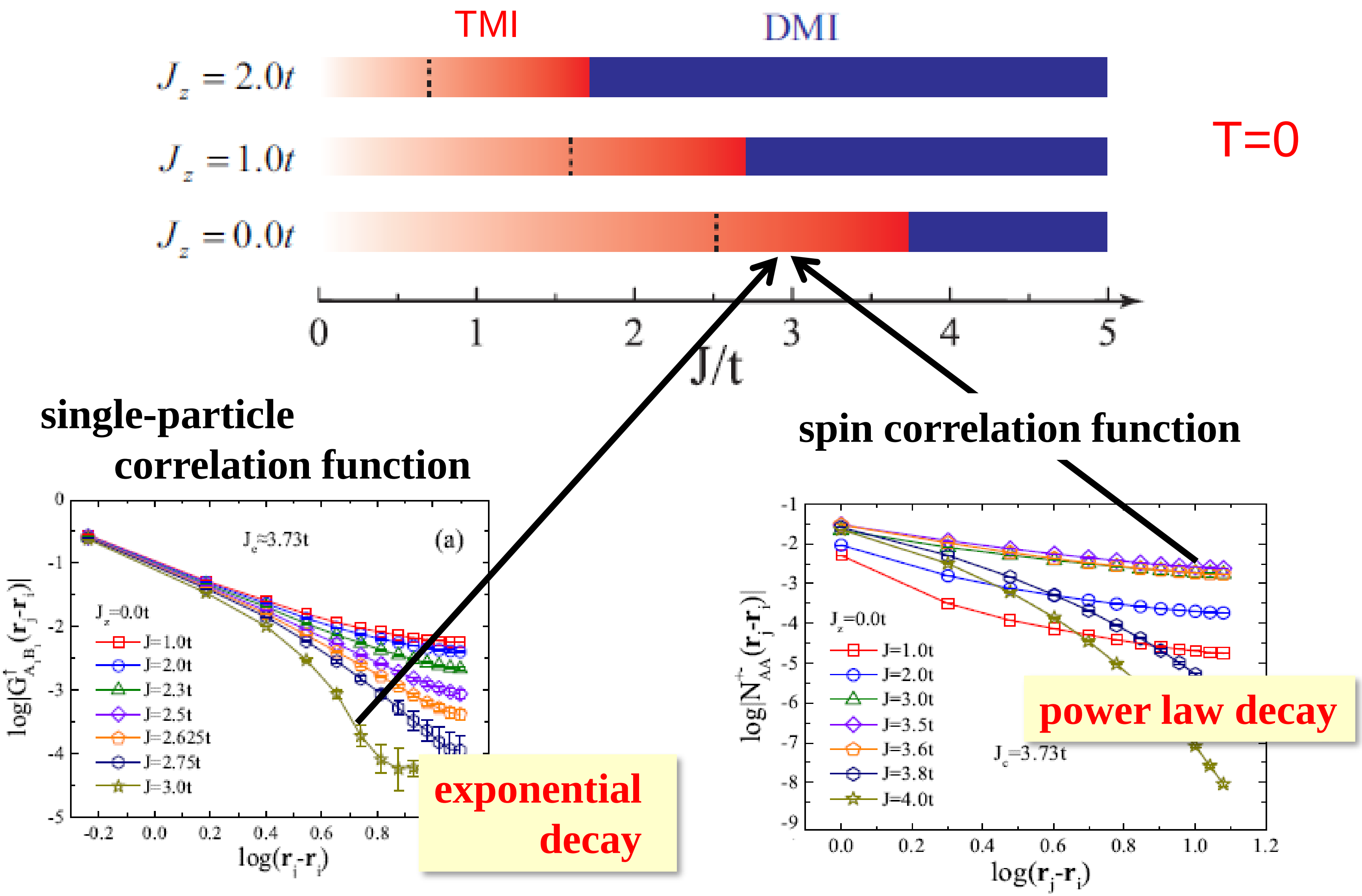
With changing T (or U)

- Topology : does not change $\sigma^{xy}_{spin} \sim 2(\frac{e}{2\pi})$
- **Statistics of edge modes changes**
  Fermi ←→ Bose

Correlation + Topology

↑ enrich

Temperature



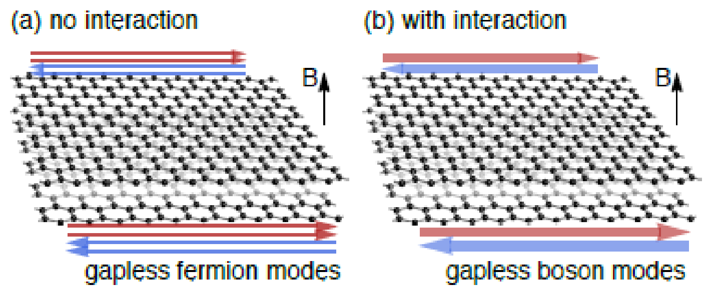
Unbiased calculations
Projector QMC T=0

$$N^{+-}_{AA}(\mathbf{r}_j - \mathbf{r}_i) = \frac{1}{2}[S^{\pm}_{A_1A_1}(\mathbf{r}_j - \mathbf{r}_i) - S^{\pm}_{A_1A_2}(\mathbf{r}_j - \mathbf{r}_i) - S^{\pm}_{A_2A_1}(\mathbf{r}_j - \mathbf{r}_i) + S^{\pm}_{A_2A_2}(\mathbf{r}_j - \mathbf{r}_i)]$$

(H-Q. Wu, Y-Y. He, Y-Z You, T.Yoshida., N.K., C. Xu, Z-Y. Meng, and Z-Y. Lu (2016)

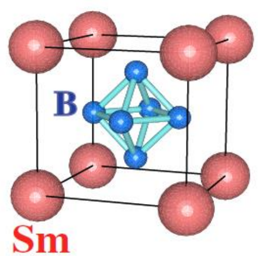
Platforms of 2D Topological Mott Insulators

1. Double layer graphen with B (repulsive interaction)



Z. Bi et al. arXiv:1602.03190
PRL(2017)

2. Kondo insulator SmB_6 thin layer



R.-X. Zhang et al. arXiv:1607.0607

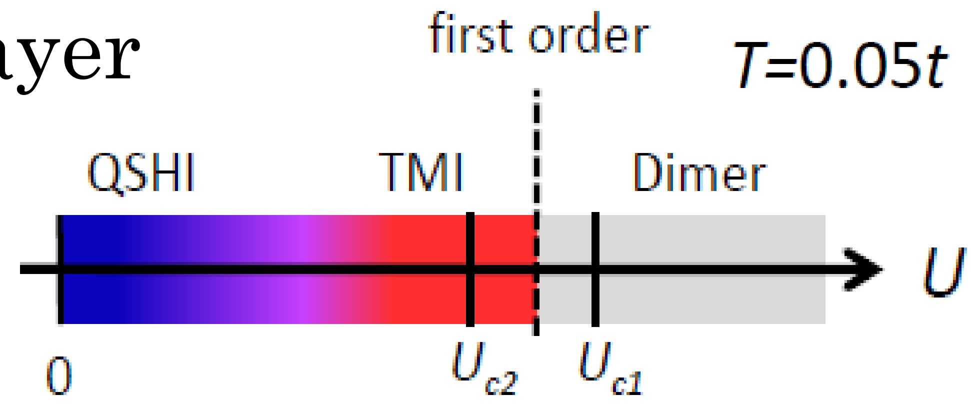
3. Cold atomic systems

Topological Haldane model was already realized (honeycomb)

Esslinger group, Nature 515 (2014)

Summary of 2D bilayer

T=0.05t



DMFT+CTQMC

- Topological Mott insulator

Crossover in temperature

Gapless edge modes

fermionic

bosonic

Topology + Correlation

T. Yoshida and NK (2016)

H-Q. Wu, Z-Y. Meng, T. Yoshida, NK et al (2016)

Reduction of Topological Classification
~ an experimental test bed ~

T. Yoshida, A. Daido, A. Yanase, NK (2017)

Reduction of topological classification by correlations

addressed by many groups.

Y.-M Lu and A. V. Vishwanath (2012);
M. Levin and A. Stern (2012);
H. Yao and S. Ryu (2013);
S. Ryu and S.-C. Zhang (2012);
C. Wang, A. C. Potter, and T. Senthil (2014);
C.-T. Hsieh, T. Morimoto, and S. Ryu (2014);
Y.-Z. You and C. Xu (2014);
H. Isobe and L. Fu (2015);
T. Morimoto, A. Furusaki, and C. Mudry (2015)
X.Y. Song and A.P. Schnyder (2017)

Periodic table in correlated systems
is obtained in 1, 2, and 3D

Symmetry class $U(1)$ only (A)	Reduction of free-fermion in 3D
AII	$\mathbb{Z}_2 \to \mathbb{Z}_2$
AI	0
AIII	$\mathbb{Z} \to \mathbb{Z}_8$
CII	$\mathbb{Z}_2 \to \mathbb{Z}_2$
DIII	$\mathbb{Z} \to \mathbb{Z}_{16}$
CI	$\mathbb{Z} \to \mathbb{Z}_4$

Wang and Senthil (2014)

Reduction of topological classification by correlations

Theory on the reduction has been advanced recently.

But...

No candidate materials for confirming the reduction of classification

We propose

$CeCoIn_5/YbCoIn_5$ superlattice as a candidate material

Experimental observations

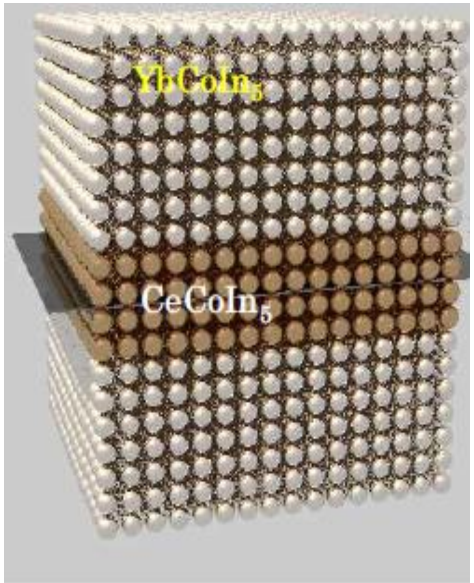
Correlated electrons are confined in

$\mathrm{CeCoIn_5}$ -layers

superconducting phase at T~1K

Y. Mizukami, *et al.*, (2011)
S.K. Goh *et al.*, (2012)
M. Shimozawa *et al.*, (2014)

reflection plane



Superlattice: topological crystalline superconductor

We find

# of $\mathrm{CeCoIn_5}$ layers	# of Majorana	protection
2	4	yes
3	1	yes
4	8	NO

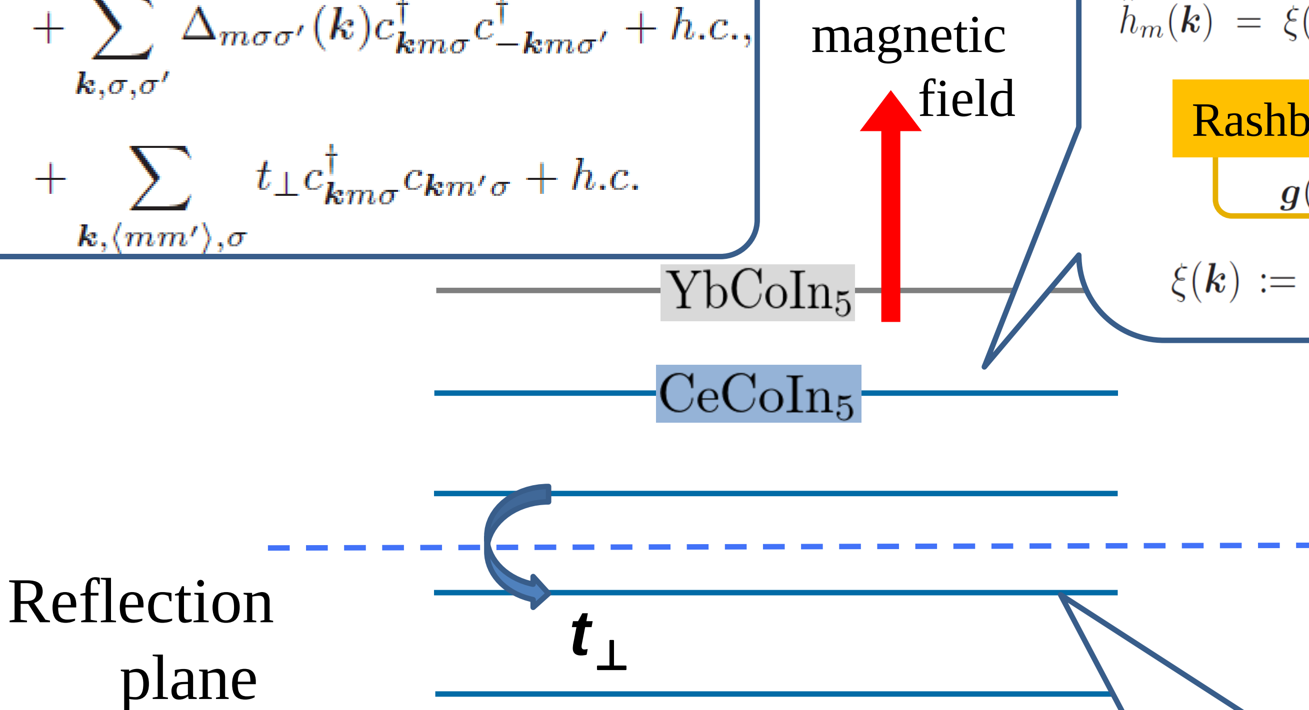
Correlation

Quad-layer superlattice: a candidate for the reduction $\mathbb{Z}\times\mathbb{Z} \rightarrow \mathbb{Z}\times\mathbb{Z}_8$

Non-interacting case: BdG-Hamiltonian with magnetic field

BdG-Hamiltonian for CeCoIn5 layers

$$H = \sum_{\boldsymbol{k},m,\sigma,\sigma'} c^\dagger_{\boldsymbol{k}m\sigma}[\hat{h}_m(\boldsymbol{k})]_{\sigma\sigma'} c_{\boldsymbol{k}m\sigma'} + \sum_{\boldsymbol{k},\sigma,\sigma'} \Delta_{m\sigma\sigma'}(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}m\sigma} c^\dagger_{-\boldsymbol{k}m\sigma'} + h.c., + \sum_{\boldsymbol{k},\langle mm'\rangle,\sigma} t_\perp c^\dagger_{\boldsymbol{k}m\sigma} c_{\boldsymbol{k}m'\sigma} + h.c.$$



intra-layer: normal part

Zeeman term

$$\hat{h}_m(\boldsymbol{k}) = \xi(\boldsymbol{k})\sigma^0 + \alpha_m \boldsymbol{g}(\boldsymbol{k}) \cdot \boldsymbol{\sigma} - \mu_B H \sigma^z$$

Rashba term

$$\boldsymbol{g}(\boldsymbol{k}) := (-\sin(k_y), \sin(k_x), 0)^T$$

$$\xi(\boldsymbol{k}) := -2t\,(\cos(k_x) + \cos(k_y)) - \mu$$

intra-layer: pairing potential

$$\Delta_m(\boldsymbol{k}) = i\,(\psi_m(\boldsymbol{k}) - \boldsymbol{d}_m(\boldsymbol{k}) \cdot \boldsymbol{\sigma})\,\sigma^y$$

$d_{x^2-y^2}$ $\quad$ $p_y - ip_x$

T. Yoshida, M. Sigrist, and Y. Yanase, PRL (2015).

Chern numbers in the superconducting phase

Block-diagonalize with reflection

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_+ & \\ & \mathcal{H}_- \end{pmatrix}$$

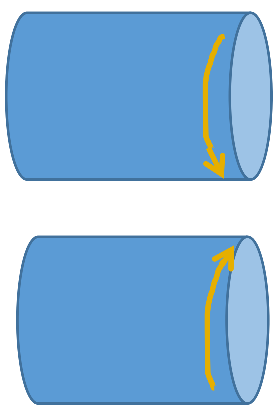
$\mathcal{H}_\pm$ is cheractrized by Chern#

→ $\mathbb{Z}^2$-classification

OBC

ν_+ ×8

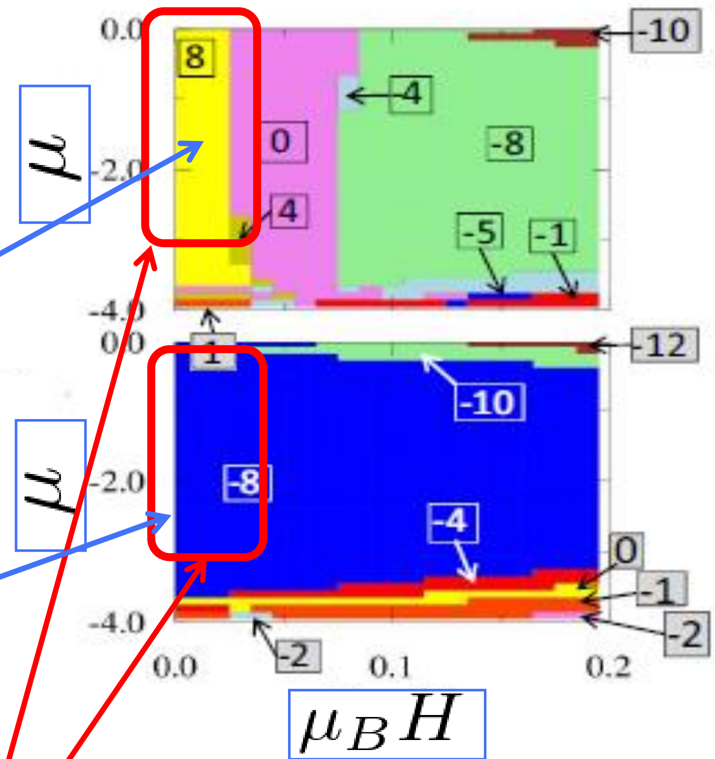
ν_- ×8



PBC: Chern number $\nu_\pm$

ν_+

ν_-



mirror Chern # $\nu_{\mathrm{M}} = \frac{\nu_+ - \nu_-}{2}$

total Chern # $\nu_{\mathrm{tot}} = \nu_+ + \nu_-$

Topological crystalline superconductor

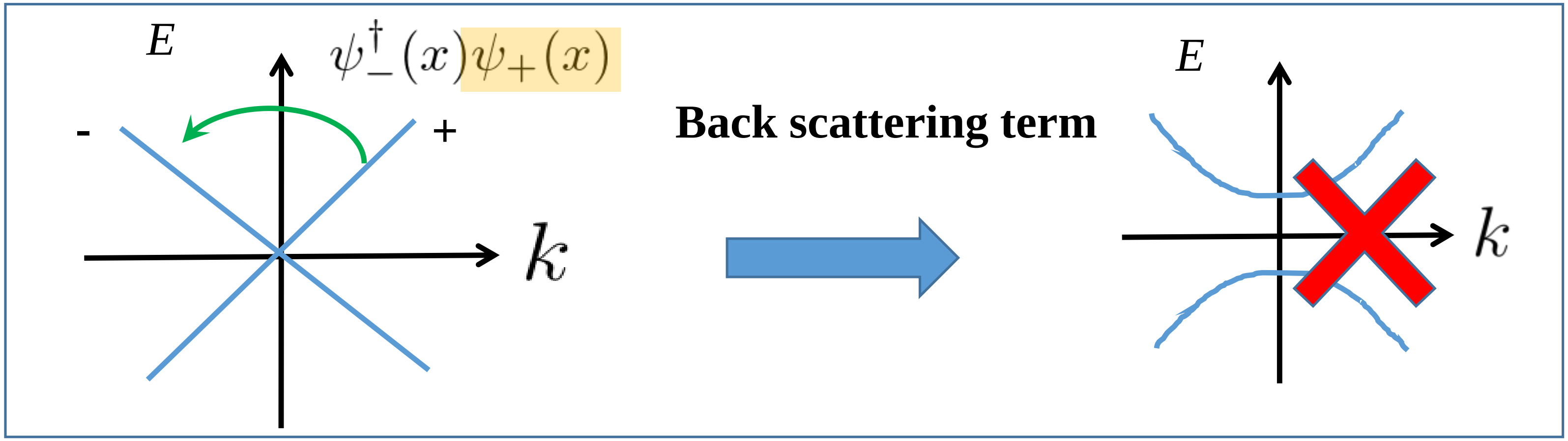
$\nu_{\mathrm{M}} = 8 \quad \nu_{\mathrm{tot}} = 0$

Gapping out respecting *R*-symmetry

two pairs of Majorana → **complex fermion**

$$\psi_\pm(x) := \eta_{1\pm}(x) + i\eta_{2\pm}(x) \qquad R\begin{pmatrix}\psi_+(x)\\ \psi_-(x)\end{pmatrix}R^{-1} = \begin{pmatrix}-\psi_+(x)\\ \psi_-(x)\end{pmatrix}$$

Free helical $H_0 = \int dx[v_F\psi_+^\dagger(x)\partial_x\psi_+(x) - v_F\psi_-^\dagger(x)\partial_x\psi_-(x)]$



$\psi_-^\dagger(x)\psi_+(x)$ breaks R-symmetry

↓

Symmetry protected gapless modes

# of helical complex fermions	Symmetry protection
1	Yes
2	Yes
3	Yes
4	**NO**

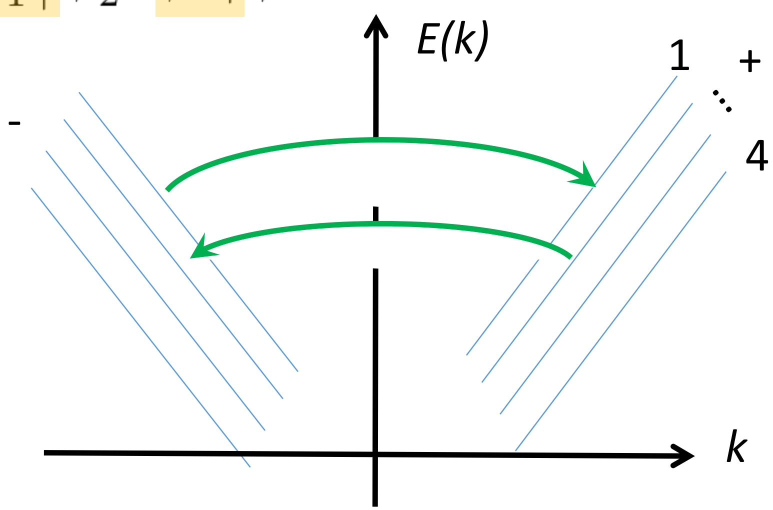
8 pairs of helical Majorana

$$R \begin{pmatrix} \psi_+(x) \\ \psi_-(x) \end{pmatrix} R^{-1} = \begin{pmatrix} -\psi_+(x) \\ \psi_-(x) \end{pmatrix}$$

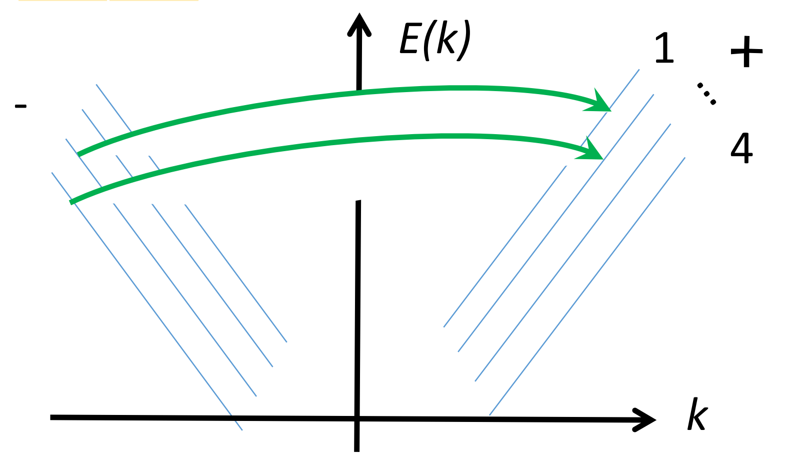
$$H_{\rm int} = \int dx[\psi^\dagger_{1+}(x)\psi^\dagger_{2-}(x)\psi_{2+}(x)\psi_{1-}(x) + h.c.,]$$
$$+ \int dx[\psi^\dagger_{3+}(x)\psi^\dagger_{4-}(x)\psi_{4+}(x)\psi_{3-}(x) + h.c.,]$$
$$+ \int dx[\psi^\dagger_{1+}(x)\psi^\dagger_{2+}(x)\psi_{3-}(x)\psi_{4-}(x) + h.c.,]$$
$$+ \int dx[\ \ldots \ .,]$$

Gap opens
Reduction occurs !

$$\psi^\dagger_{1+}\psi^\dagger_{2-}\psi_{2+}\psi_{1-} + h.c.$$



$$\psi^\dagger_{1+}\psi^\dagger_{2+}\psi_{3-}\psi_{4-} + h.c.$$

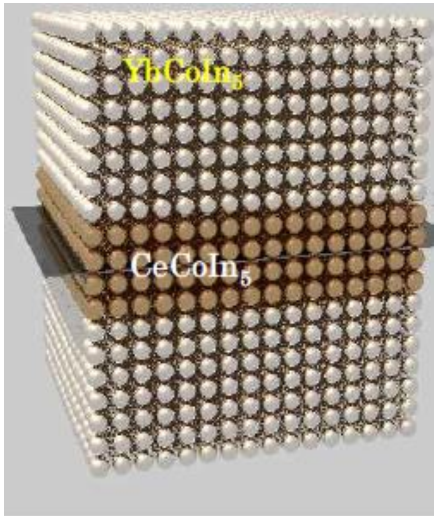


Testbed for reduction of Topo classification

We propose the $CeCoIn_5/YbCoIn_5$ superlattice system as a test bed of reduction of topological classification:

$$\mathbb{Z}\times\mathbb{Z}\longrightarrow\mathbb{Z}\times\mathbb{Z}_8$$

# of $CeCoIn_5$ layers	$(\nu_{\mathrm{M}},\nu_{\mathrm{tot}})$	# of Majorana	Protection (correlated)
2	(4,0)	4	yes
3	(1,0)	1	**yes**
4	(8,0)	8	**NO**



This might be observed with systematic STM measurement for 2,3,4,5,6,…layers

Summary

Correlation Effects in Topological Insulators/Superconductors

1. Topological Mott Insulator

- Topological Mott insulator
- Edge Mott states 1D & 2D

- T-induced change in Fermi-Bose statistics 2D

2. Reduction of Topological Classification

$CeCoIn_5/YbCoIn_5$ superlattice
a testbed for reduction of topological classification