Many-body localization in disordered spin and Hubbard chains

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Hvar, October 3, 2017

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Jacek Herbrych, Knoxville

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M. Mierzejewski, M. Kozarzewski, P. Prelovšek, arXiv (2017)

Many-body localization: goal

Create a macroscopic quantum MB system which does not thermalize at any temperature and retains the information locally ?

Ideal nonequilibrium system = absence of thermalization

no d.c. transport at any T !

nonergodicity of (all) correlations

local quantitites : qubits

no leakage of quantum information

Two extremes: integrable systems – ideal conductivity at $T > 0$

MBL – no transport at $T > 0$

Outline

Why MBL is so fascinating ?

Experiments on MBL: cold atoms, spin chains ?

Characteristic features of MBL systems:

- numerical results on the 'standard' model of MBL
- vanishing d.c. transport and dynamical conductivity
- non-ergodic behaviour of correlation functions

Is there MBL in 1D disordered Hubbard model ?

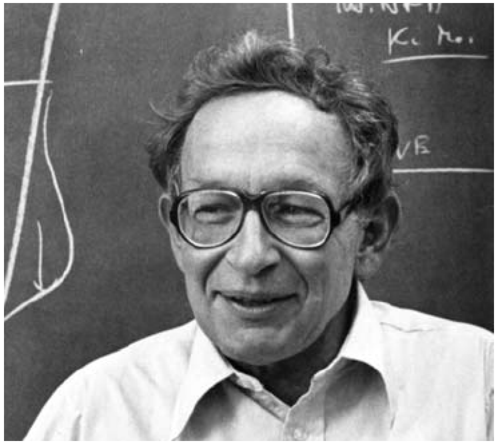
- decay of charge and spin correlations different
- no full MBL

Counting local integrals of motion (LIOM)

Anderson localization

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)



$$H = -t \sum_{\langle ij \rangle} (c_j^\dagger c_i + \mathrm{h.c.}) + \sum_i \epsilon_i c_i^\dagger c_i$$

single – particle problem

random, uncorrelated

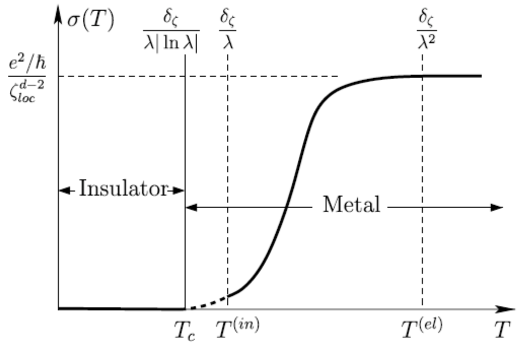
Very few believed [localization] at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it.

—Philip W. Anderson, Nobel lecture,
8 December 1977

What is MBL and why is it so interesting ?

Nonergodic behaviour in a macroscopic MB quantum system: T > 0

- non-interacting (NI) fermions on disordered lattice: Anderson localization
- integrable MB models: Heisenberg chain etc…
- systems undergoing phase transition (macroscopic ordering at $T < T_c$)
- **many-body-localized systems** = correlations + large disorder ?



Basko, Aleiner, Altshuler (2006):

- MI transition at T=T* at fixed disorder W
- MI transition at $W=W_c$ even at $T = \infty$!

> 600 theoretical papers after 2006 > 100 papers / year

Does MBL exist (phase transition or crossover ..) ?

Which are properties of the ergodic and non-ergodic phase ?

'Standard' model of MBL

1D isotropic (or anisotropic) Heisenberg model + random fields:

$$H = J \sum_i \left[\frac{1}{2}(S_{i+1}^+ S_i^- + S_{i+1}^- S_i^+) + \Delta S_{i+1}^z S_i^z \right] + \sum_i h_i S_i^z$$

$$h_i \in [-W, W]$$

Jordan – Wigner transformation (1D)

equivalent to disordered chain of interacting spinless fermions

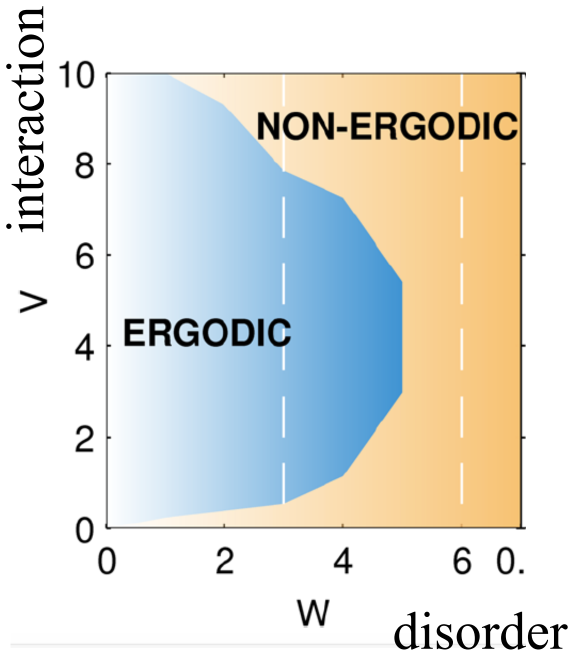
= Anderson model + interaction

$$t = J/2$$

$$H = -t \sum_i (c_{i+1}^\dagger c_i + \text{h.c.}) + \sum_i h_i n_i + V \sum_i n_{i+1} n_i$$

$$V = J\Delta$$

T ~ ∞: phase diagram (approximate ?)



$$H = -t\sum_i (c_{i+1}^\dagger c_i + \mathrm{h.c.}) + \sum_i h_i n_i + V\sum_i n_{i+1} n_i$$

Bar Lev et al, PRL (2015)

ergodic phase: W < W_c (V)

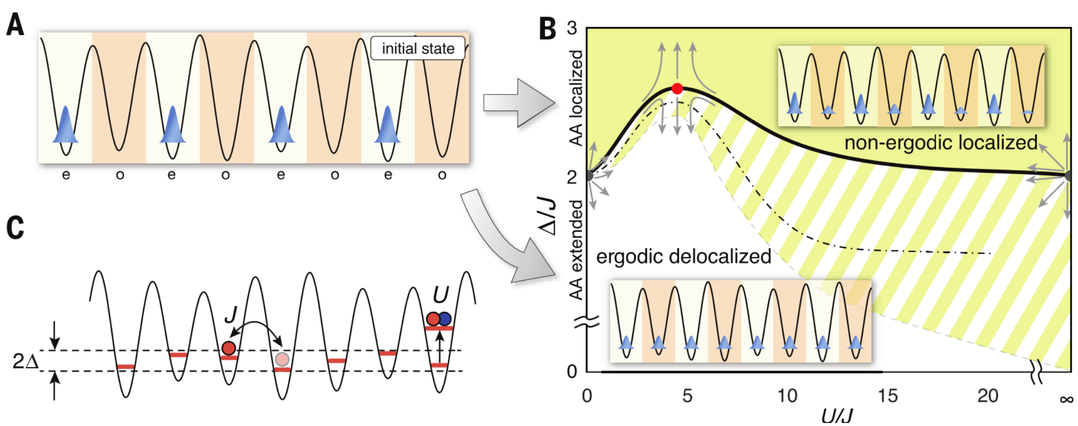
nonergodic (MBL) phase: W > W_c (V)

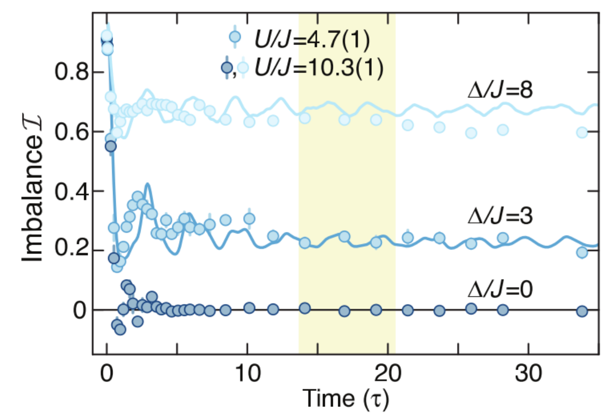
W > W_c:

- Poisson MB level statistics
- vanishing d.c. transport – spin (particle) , energy
- area (log) law for entanglement entropy increase
- non-ergodic behaviour of (all) correlation functions, no thermalization
- **local integrals of motion**

Cold atoms (fermions) on 1D optical lattice

Schreiber et al, Science (2015): K^{40} atoms on 1D optical lattice
\+ quasi-periodic (Andre-Aubry) disorder



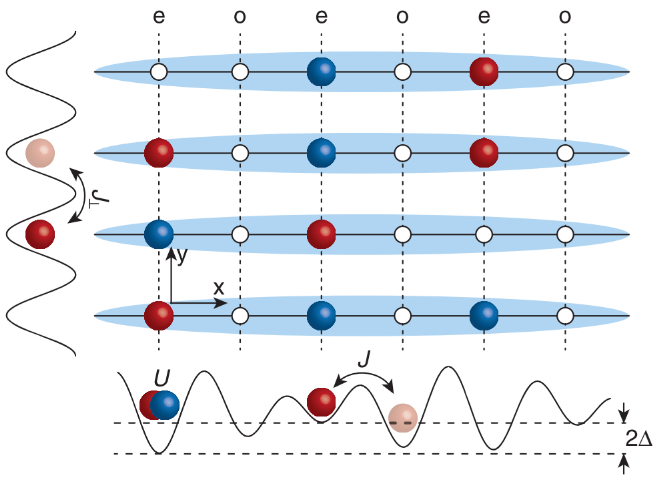


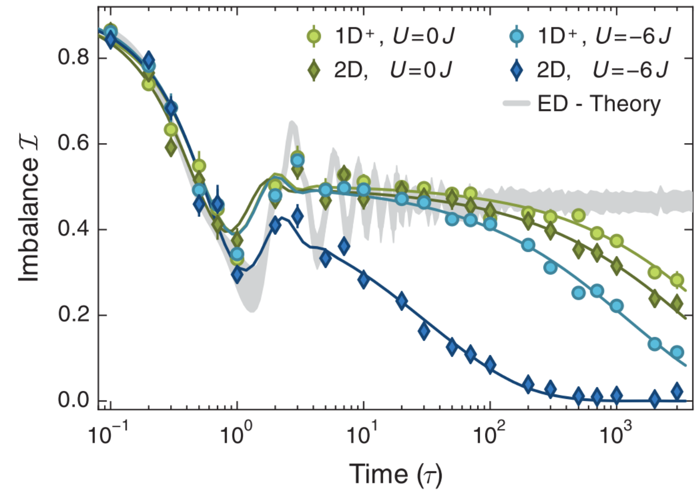
$$\hat{H} = -J\sum_{i,\sigma}(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + \text{h.c.}) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

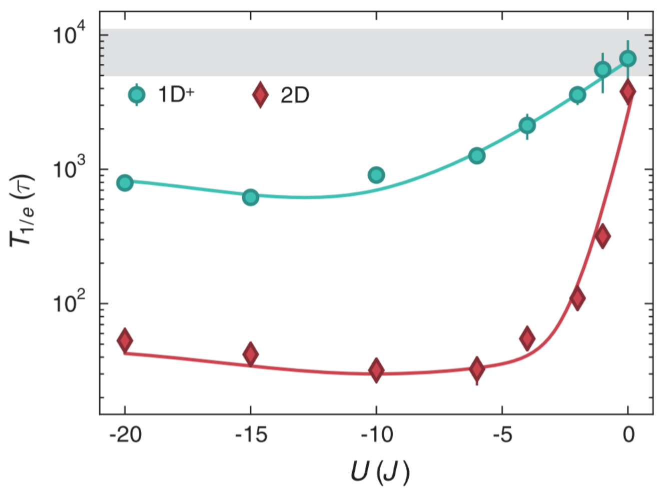
(charge) density **imbalance I(t)**:
non-ergodic for large disorder Δ

model: 1D Hubbard model with quasi-periodic (random) potential

Bordia et al, PRL (2016): interacting fermions on coupled (Hubbard) 1D chains
+ identical (Andre-Aubry) disorder on all chains



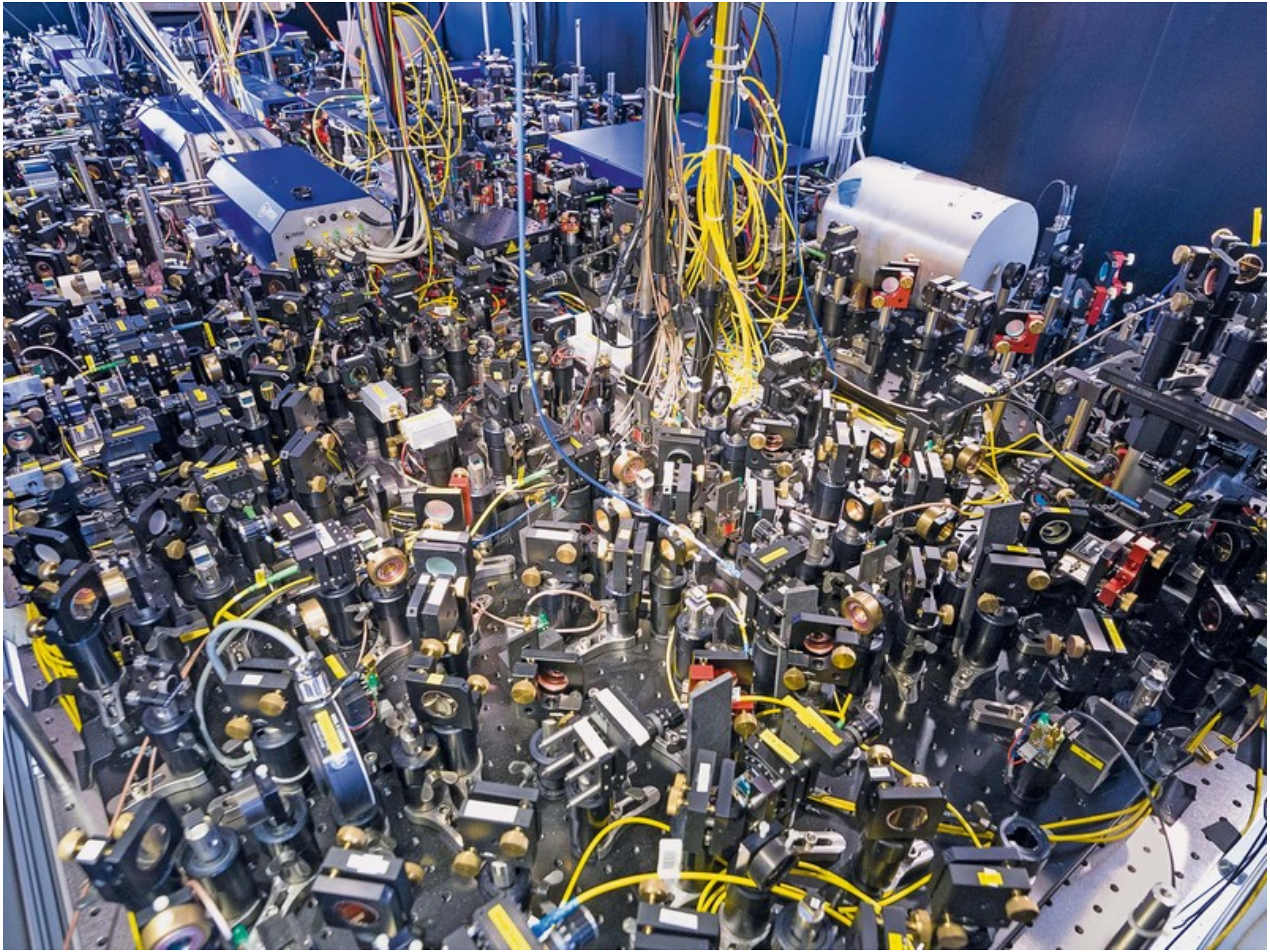




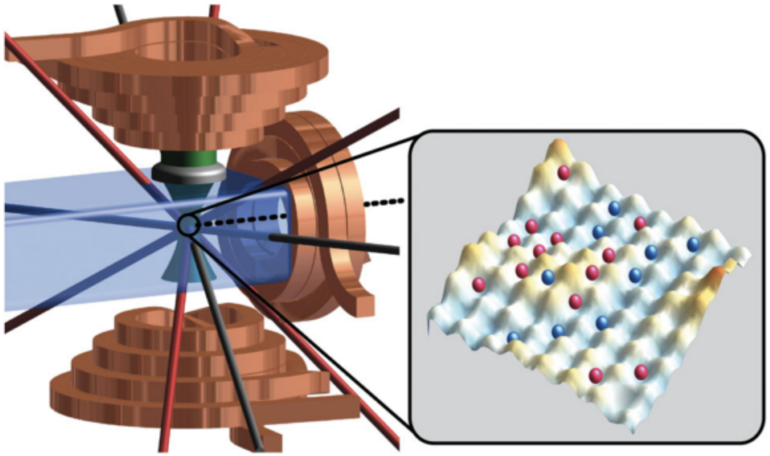
(C)DW decay only for |U| > 0
+ transverse coupling

experiments are on effective
Hubbard model !

LMU, MPQE , München, A. Giersch

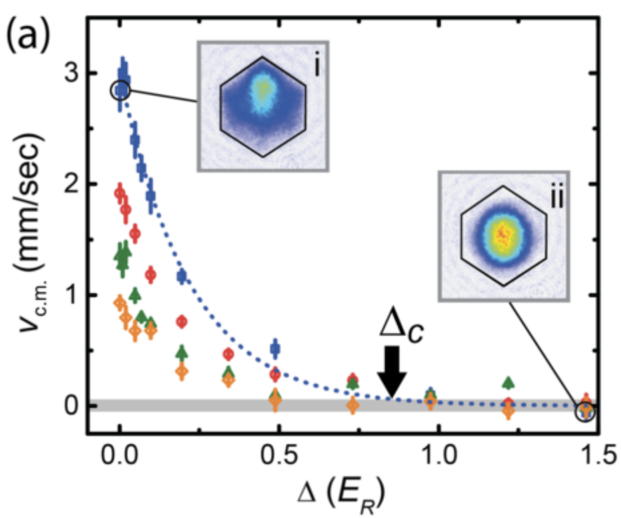


Cold atoms on 3D random lattice



Kondov et al, PRL (2015)

3D lattice + random potential



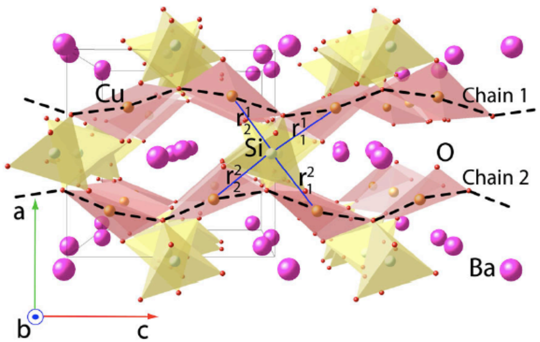
force via magnetic field gradient:

center-of-mass velocity

MBL localization : disorder $\Delta > \Delta_c$

Can MBL appear in real materials ?

random exchange Heisenberg chain: S=1/2



Shiroka et al. PRL (2011)

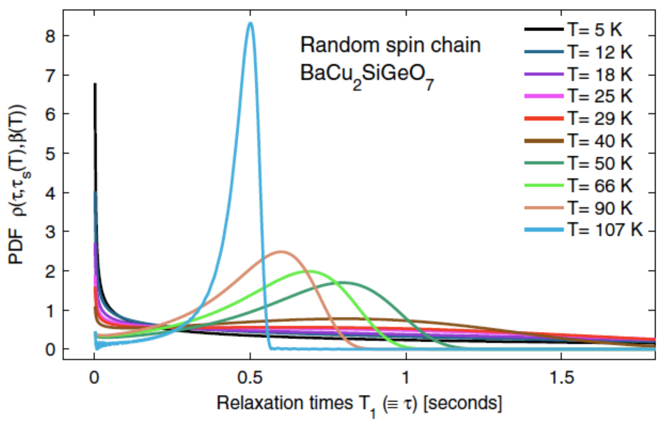
$BaCu_2(Si_{1-x}Ge_x)_2O_7$

$J_{Si} = 280\,K \qquad J_{Ge} = 580\,K$

NMR magnetization recovery:

pure system: exponential decay
random system: stretched exponential

$$\frac{M_z(t)}{M_0} \sim 1 - \exp\left[-(t/\tau_0)^{\Gamma}\right]$$

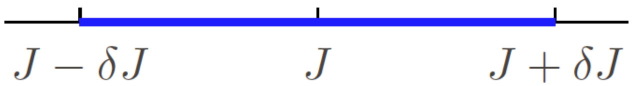


distribution of relaxation rate very broad, **even singular at T ~ 0** ?

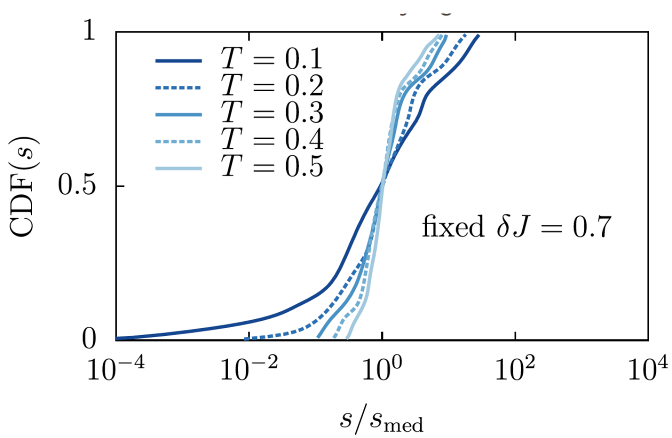
random exchange Heisenberg chain: S=1/2

$$H = \sum_i J_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right)$$

Herbrych et al., PRL (2013)



distribution of $1/T_1$ local relaxation



very broad distribution at T ~ 0

dynamical spin structure factor: S(q,ω)

no spin diffusion at T > 0 ?

indication for MBL ??

but no random field, so SU(2) symmetry !

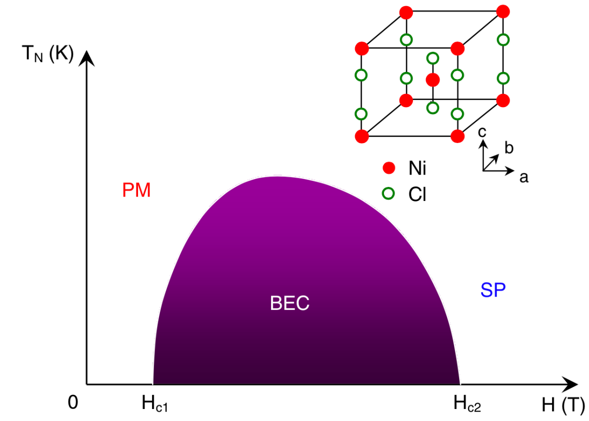
$$\mathrm{CDF}(x) = \int_0^x \mathrm{d}y \, \mathrm{PDF}(y)$$

random S=1 chain: in external field

$$H = \sum_{i=1}^{L} \left[J_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + D_i (S_i^z)^2 + h S_i^z \right]$$

doped DTN: $Ni(Cl_{1-x}Br_x)_2$-$4SC(NH_2)_2$

Herbrych, Kokalj, PRB (2017)



DTN: $NiCl_2$-$4SC(NH_2)_2$

$h \sim D$: large uniform magnetic field – **mapping on S=1/2 model**

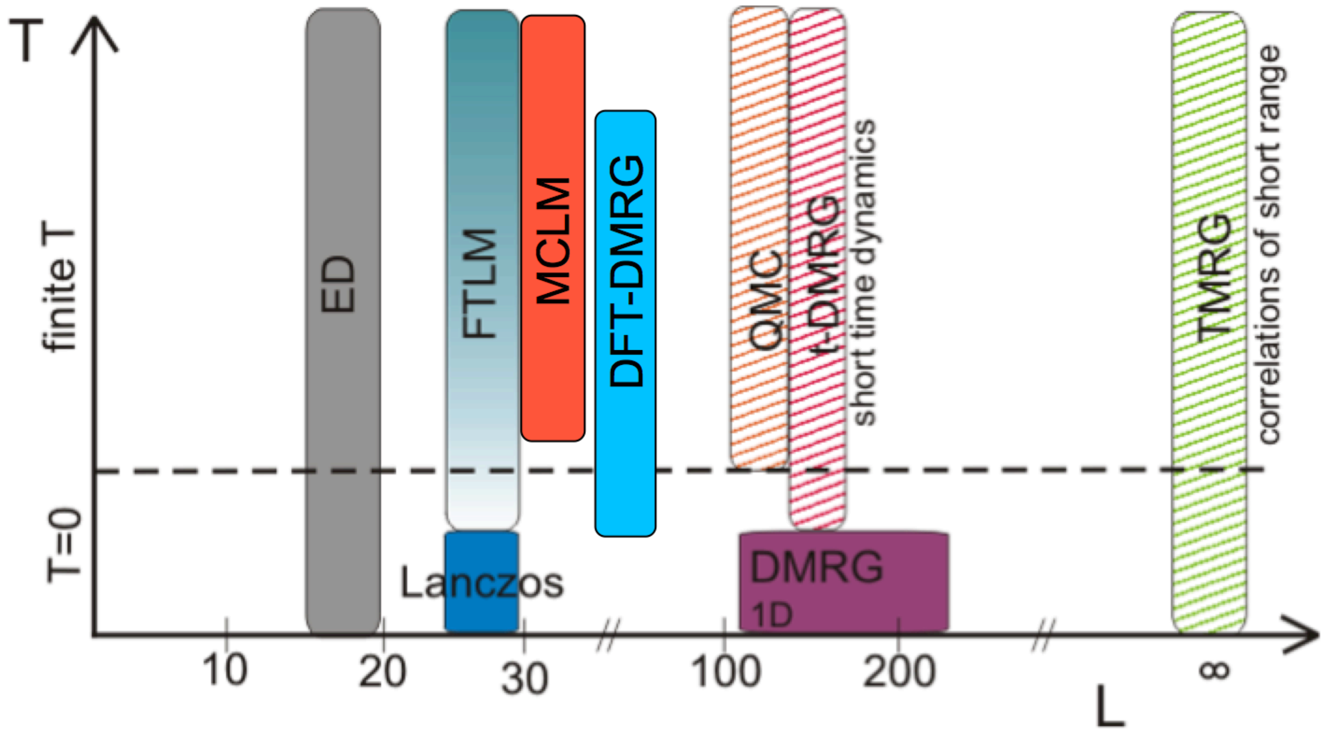
$$\widetilde{H} = \sum_{i=1}^{L} \left[\widetilde{J}_i \left(\widetilde{S}_i^x \widetilde{S}_{i+1}^x + \widetilde{S}_i^y \widetilde{S}_{i+1}^y + \Delta \widetilde{S}_i^z \widetilde{S}_{i+1}^z \right) + \widetilde{h}_i \widetilde{S}_i^z \right] \qquad \Delta = 0.5$$

with random effective field ! $\delta\widetilde{h} = \delta J + \delta D$

in doped DTN the field randomness too small for MBL at T = ∞ ?

but possibly not for T > 0 ??

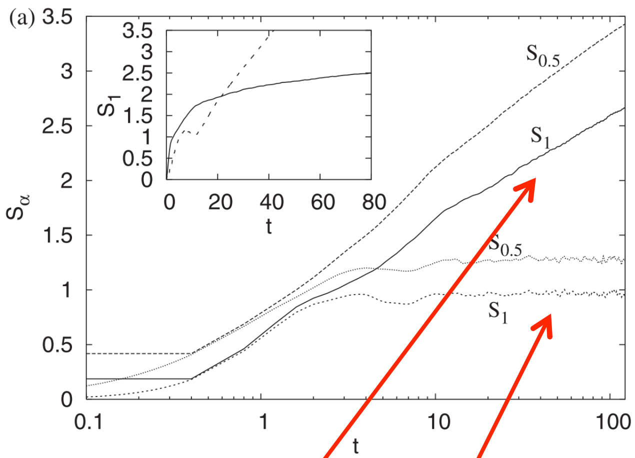
Numerical methods for MBL (dynamics)



MBL numerical problem: **T >> 0 + very long times - low ω !**

+ large sizes ?

Characteristic features: entanglement entropy



Žnidarič, Prosen, PP, PRB (2008)

bipartite entanglement: subsystems A + B

$$\rho_A = \mathrm{tr}_B |\psi(t)\rangle\langle\psi(t)|$$

$$S_1(t) = \mathrm{tr}(\rho_A \log_2 \rho_A)$$

von Neumann entropy

$S_1(t)$ saturates for NI ($\Delta = 0$) system

$S_1(t)$ increases logarithmically for MBL ($\Delta = 0.5$) system

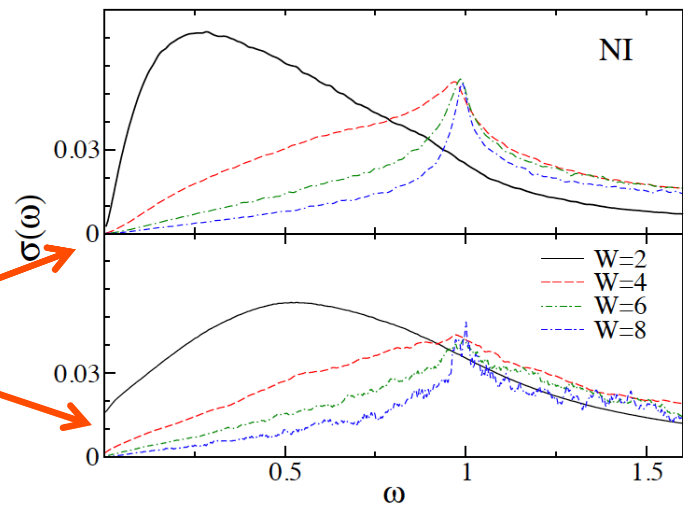
$S_1(t) = c\,t$ - linear increase for ‘normal’ system

Characteristic feature: dynamical conductivity and d.c. transport

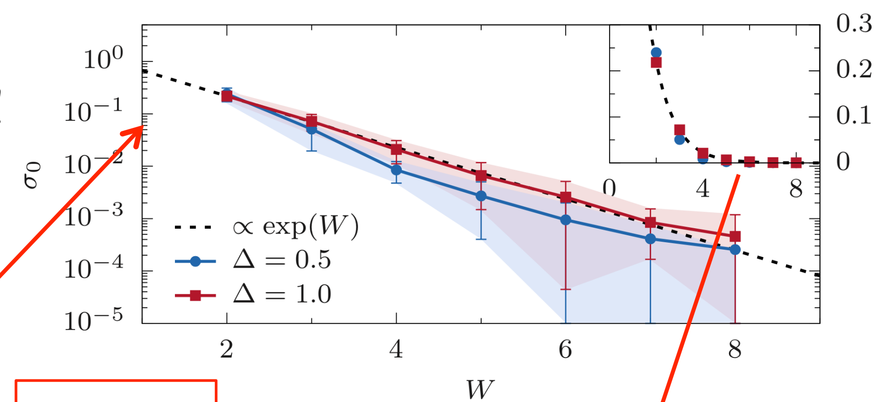
Barišić et al, PRB (2010, 2016)

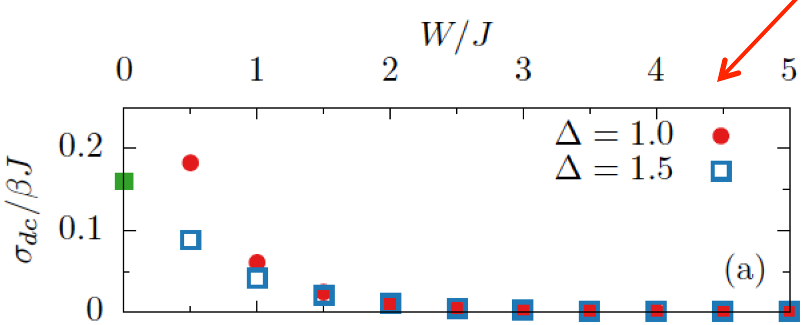
vanishing d.c. transport σ_0 for $W > W_c \sim 5$

$$\sigma(\omega) \sim \sigma_0 + \zeta|\omega|^\alpha \qquad \alpha \sim 1$$



log !



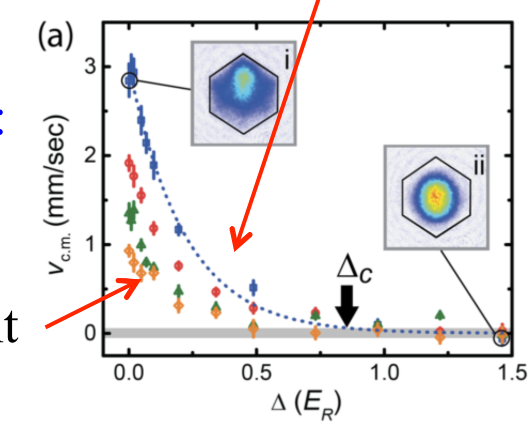


T >> 1:

sharp transition :

crossover ??

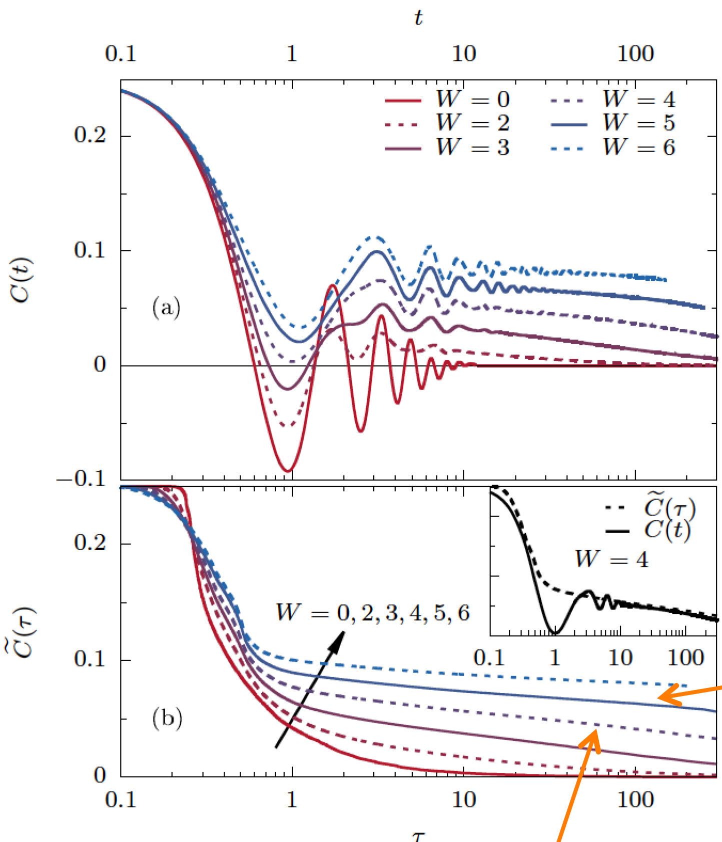
theory : experiment



Steinigeweg et al, PRB (2016)

Characteristic feature: nonergodicity and universal dynamics

Mierzejewski et al., PRB (2016)



$\widetilde{C}(\tau) \sim \log \tau / \tau^*$

density-wave (imbalance) correlation function: $T = \infty$, $V = t$ (Δ=0.5), ED, L = 16

$$C(\omega) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty \mathrm{d}t\, e^{\imath\omega^+ t}\langle n_{q=\pi}(t)\, n^\dagger_{q=\pi}\rangle$$

a) real-time dynamics: $C(t) = \int_{-\infty}^{\infty} \mathrm{d}\omega\, e^{-\imath\omega t} C(\omega)$

oscillations emerging from NI physics

b) ‘quasi’-time dynamics: $\widetilde{C}(\tau) = \int_{-1/\tau}^{1/\tau} \mathrm{d}\omega\, C(\omega)$

the same long-time variation

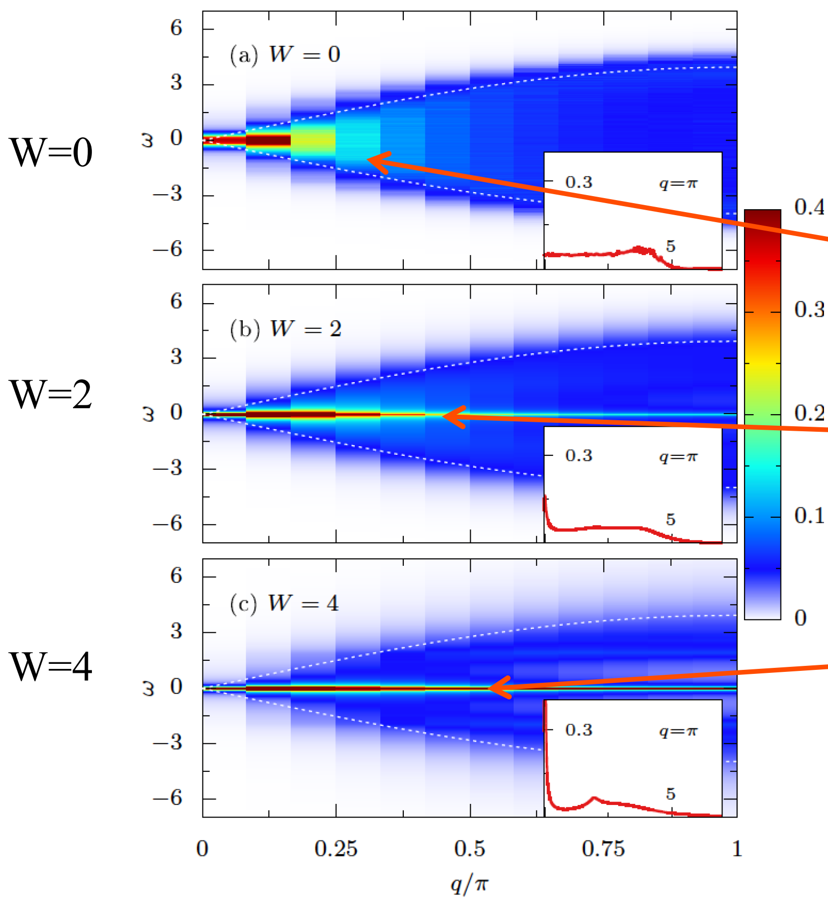
$\tilde{C}(\tau) \sim C_0 + a\, t^{-\gamma}$

nonergodic (MBL) phase: $W > W^* \sim 4$

$C_0 = C(t{=}\infty) > 0$ + anomalous time dependence $C(\omega) = C_0\delta(\omega) + C_{\mathrm{reg}}(\omega)$

Dynamical structure factor

PP, Herbrych, PRB (2017)



$$S(q,\omega) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty dt e^{i\omega t}\langle n_q(t) n_{-q}\rangle$$

MCLM: L = 26 **T = ∞, Δ = 1**

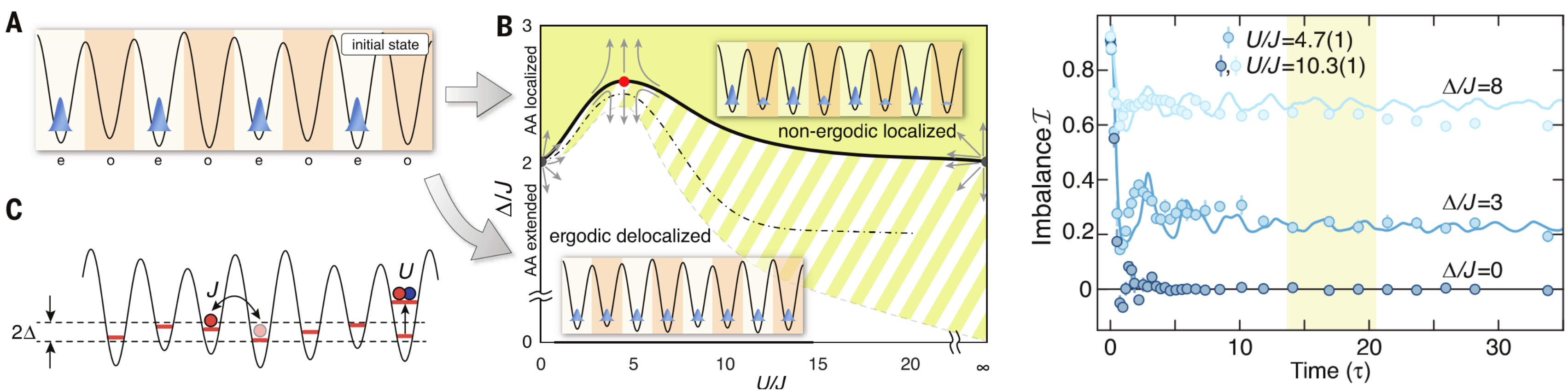
NI particles: response for W = 0

normal metal:
diffusion q ~ 0 pole for W < W_c

MBL:
delta ω ~ 0 peak for W > W_c at all q !

Cold atoms (fermions) on 1D optical lattice

Schreiber et al, Science (2015): K^{40} atoms on 1D optical lattice
+ quasi-periodic (Andre-Aubry) disorder



$$\hat{H} = -J\sum_{i,\sigma}(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + \text{h.c.}) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^\dagger_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

(charge) density imbalance I(t):
non-ergodic for large disorder Δ

model: **1D Hubbard model** with quasi-periodic (random) potential

MBL in 1D Hubbard chain

PP, Barišić, Žnidarič, PRB (2016)

potential disorder

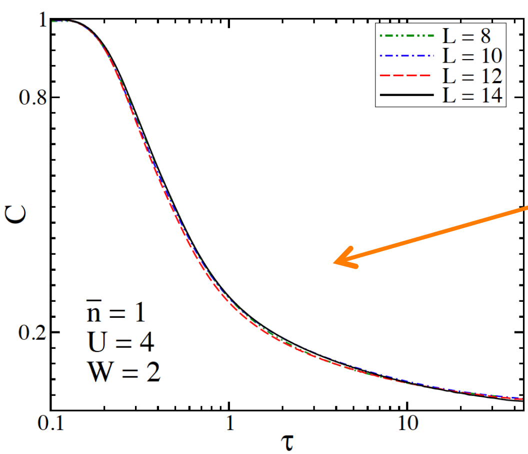
$$H = -t\sum_{is}(c^{\dagger}_{i+1,s}c_{is} + c^{\dagger}_{is}c_{i+i,s}) + U\sum_{i} n_{i\uparrow}n_{i\downarrow} + \sum_{i} h_i n_i \qquad -W < h_i < W$$

Hubbard model: more degrees of freedom – charge + spin

$n_i = n_{i\uparrow} + n_{i\downarrow}$ $m_i = n_{i\uparrow} - n_{i\downarrow}$

numerical calculation of imbalance correlations: MCLM

charge (CDW) correlations: $C(t) = \frac{\alpha}{L}\langle n_{q=\pi}(t) n_{q=\pi}\rangle$

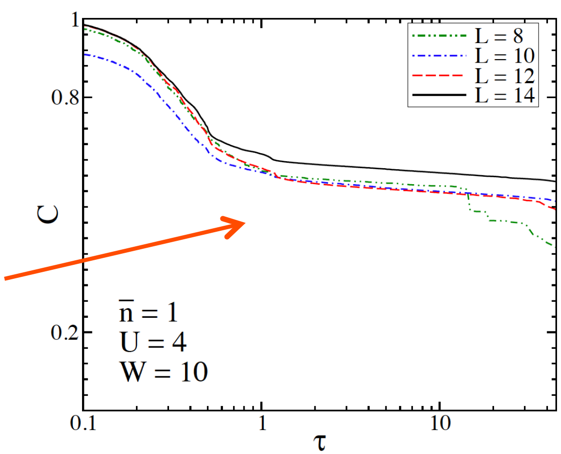


U=4, n=1, L= 8 - 14

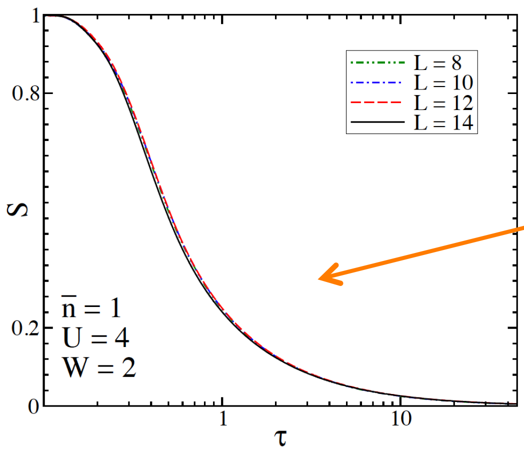
W=2: ergodic

W=10: non-ergodic

expected ?



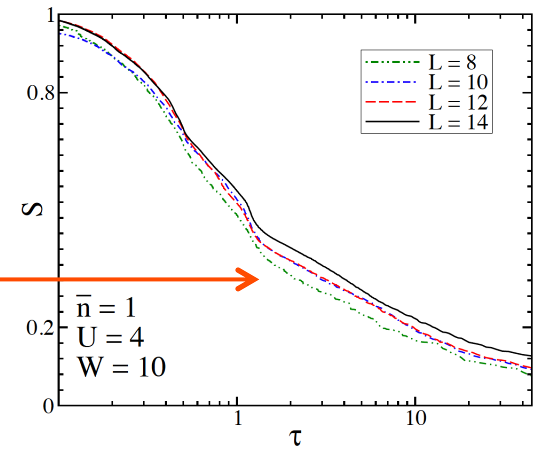
spin (SDW) correlations: $S(t) = \frac{\alpha}{L}\langle m_{q=\pi}(t) m_{q=\pi}\rangle$

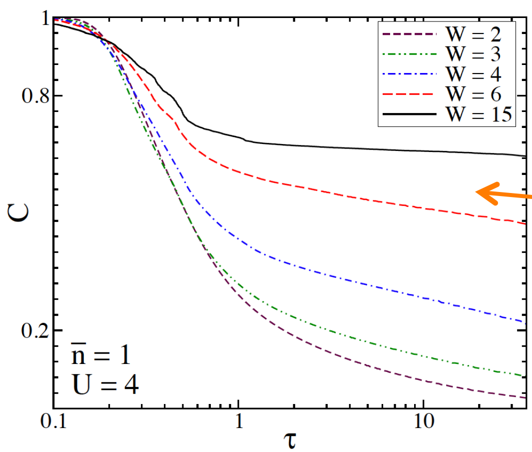


U=4, n=1, L=8-14

W=2: ergodic

W=10: ergodic ?



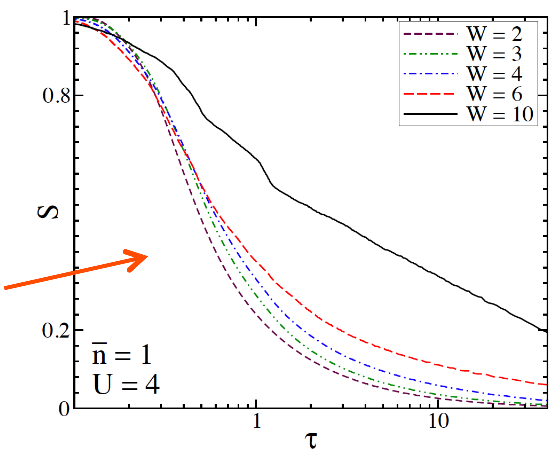


U=4, L=14

charge: $W_c \sim$ 4-6

spin: no transition

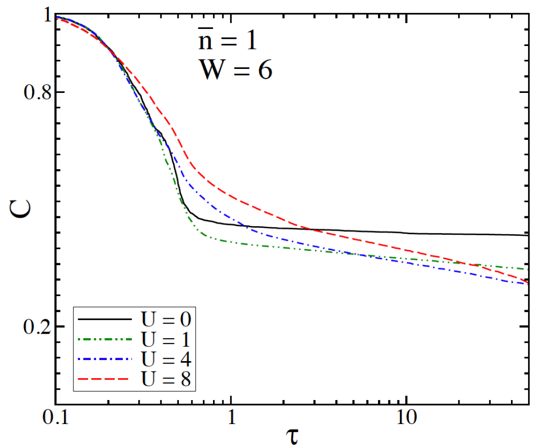
no full MBL !



varying U: from Anderson localization to MBL ?

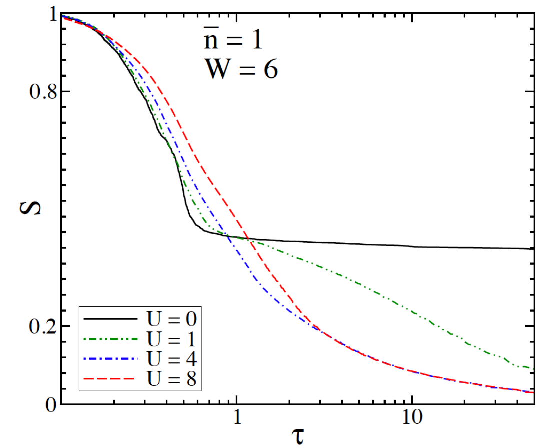
$W > W_c$: large disorder

charge



W = 6

spin



U > 0 induces weak decay of CDW,

charge localization remains

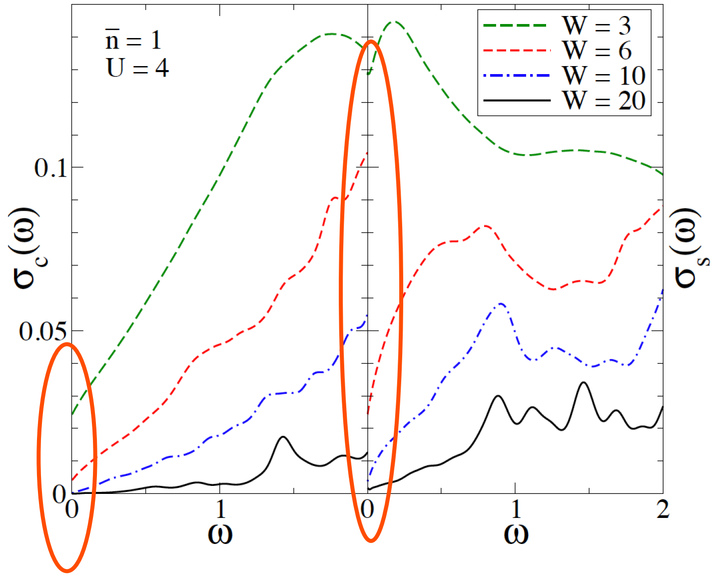
U > 0 leads to decay of SDW

spin behaves ergodic

disorder induced charge – spin separation !?

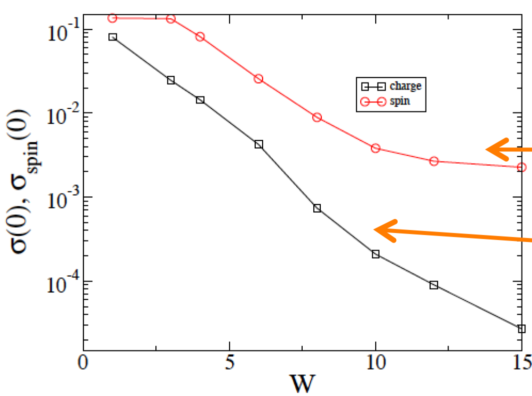
Dynamical charge and spin conductivities:

charge



spin

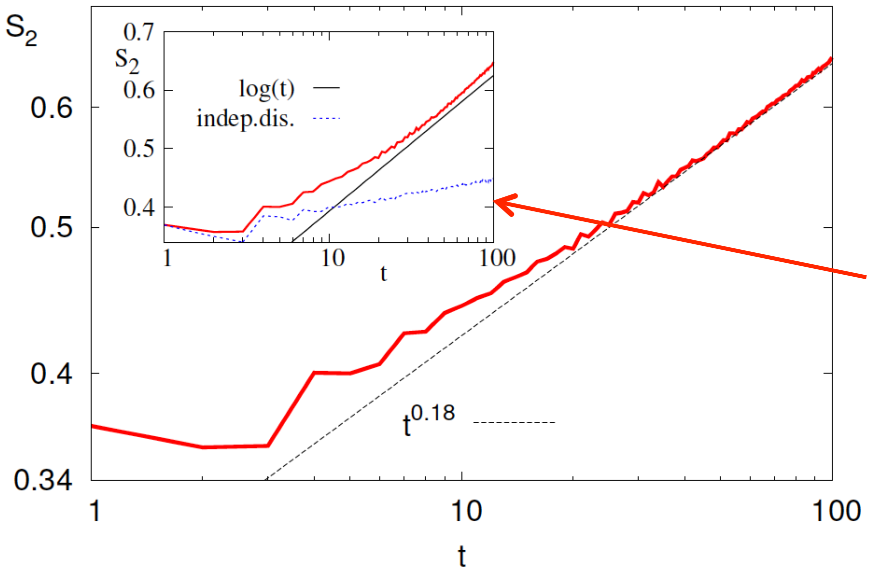
log !



d.c. charge and spin conductivity:

- $\sigma_{spin}(0)$ always finite ?
- $\sigma_{charge}(0)$ – at W ~ 4 transition or crossover ?

Entanglement entropy:



$$S_2(t) = -\mathrm{tr}[\rho_A(t)\log_2 \rho_A(t)]$$

full MBL: $S_2(t) \sim \log(t)$

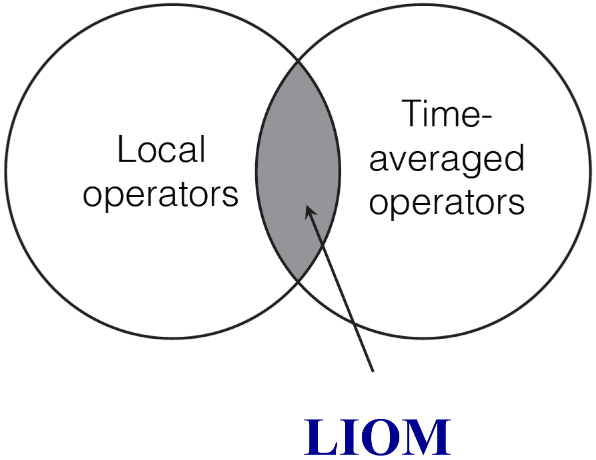
the case with random field disorder

Hubbard chain – potential disorder

$S_2(t) \sim t^{0.18}$ not full MBL !

Local integrals of motion

Mierzejewski et al., arXiv (2017)



1) local operators: create all with support M !

$$A = A_1 \otimes ... \otimes A_M \otimes \mathbb{1}_{M+1} \otimes .....\mathbb{1}_L$$

$$A = n_1 c_2^\dagger n_3 c_4, \;\; M = 4$$

2) find time-averaged ones: constants of motion !

$$\bar{A} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau \mathrm{d}t' \exp(iHt') A \exp(-iHt')$$

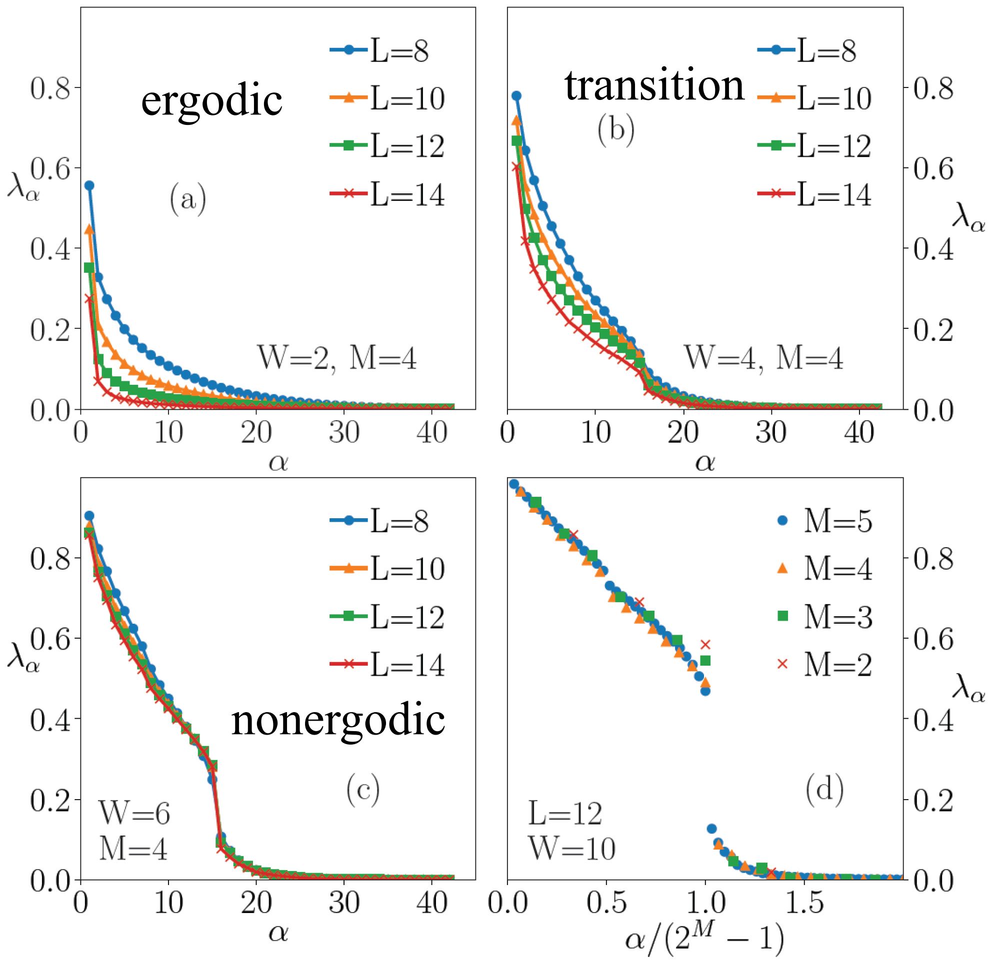
$$= \sum_m |m\rangle\langle m|A|m\rangle\langle m|. \qquad [H, \bar{A}] = 0$$

3) find all orthogonal Q_α with support M << L: $\bar{Q}_\alpha = \lambda_\alpha Q_\alpha + Q_\alpha^\perp$

$0 < \lambda_\alpha < 1 \text{ for } L \gg M$ - operator LIOM

Counting LIOMs

Disordered Heisenberg model: $N_M = 2^M - 1$ orthogonal LIOM ? Yes

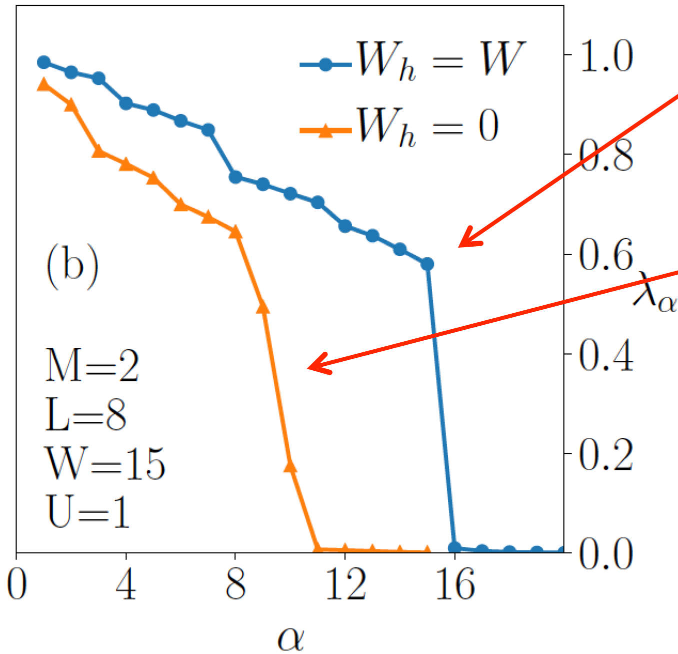


ED:

M = 4, L = 8 – 14

full MBL !

Disordered Hubbard model: $N_M = 4^M - 1$ orthogonal LIOM ? No



adding random magnetic field: full MBL

only potential disorder: not full MBL

$\tilde{N}_M \sim 3^M - 1$

Summary

‘Standard’ model of MBL:

- two regimes: ergodic $W < W_c$, non-ergodic $W > W_c$
- non-ergodic regime: vanishing d.c. transport, $C(t=\infty) > 0$
- **many open questions:** is the transition sharp or a crossover ?

Universal behaviour at the MBL transition:

- order parameter for MBL: imbalance stiffness: $C_0 = C(t=\infty) > 0$?
- better definition: universal critical dynamics - $\alpha = \zeta = 1$!

MBL in 1D disordered Hubbard chains (with potential disorder):

- CDW and SDW decay qualitatively different
- disorder induced charge – spin separation (at all energy scales) ?

Counting local integrals of motion:

- disordered Heisenberg model: full MBL – as many LIOM as local operators
- Hubbard model with potential disorder: not full MBL ?