

Advances in the theory and experiments of spin spectroscopy

Ferenc Simon

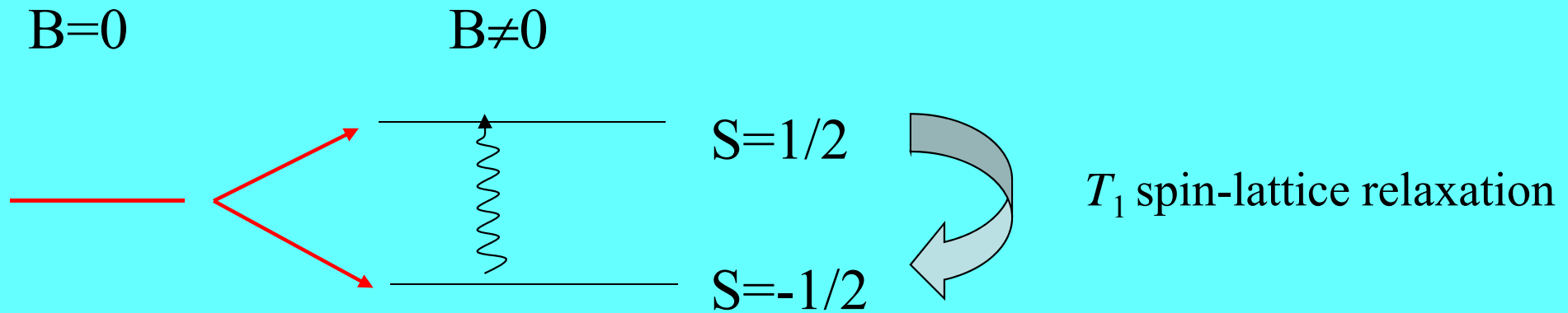
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Spin spectroscopy-Electron spin resonance



$$H_Z = g\mu_B B S \rightarrow h\nu = g\mu_B B$$

First CESR on Na: *Griswold, Kip, Kittel, PR 88, 951 (1952).*

Measurables:

Intensity

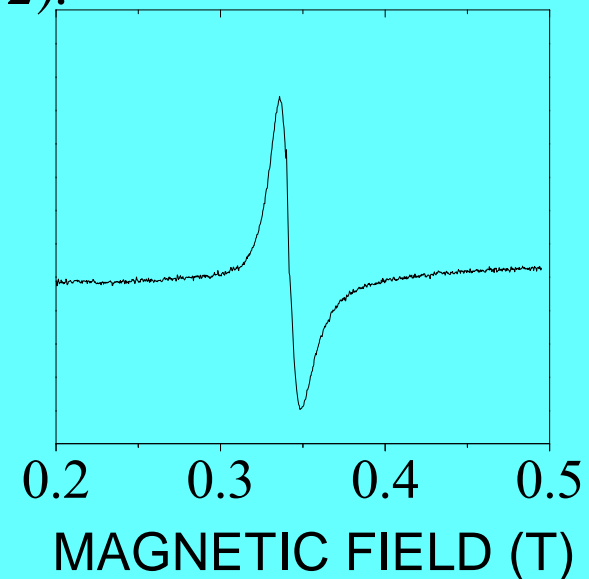
$$\chi_{\text{Pauli}} = \frac{1}{4} \mu_0 g^2 \mu_B^2 D(\epsilon_F)$$

Width

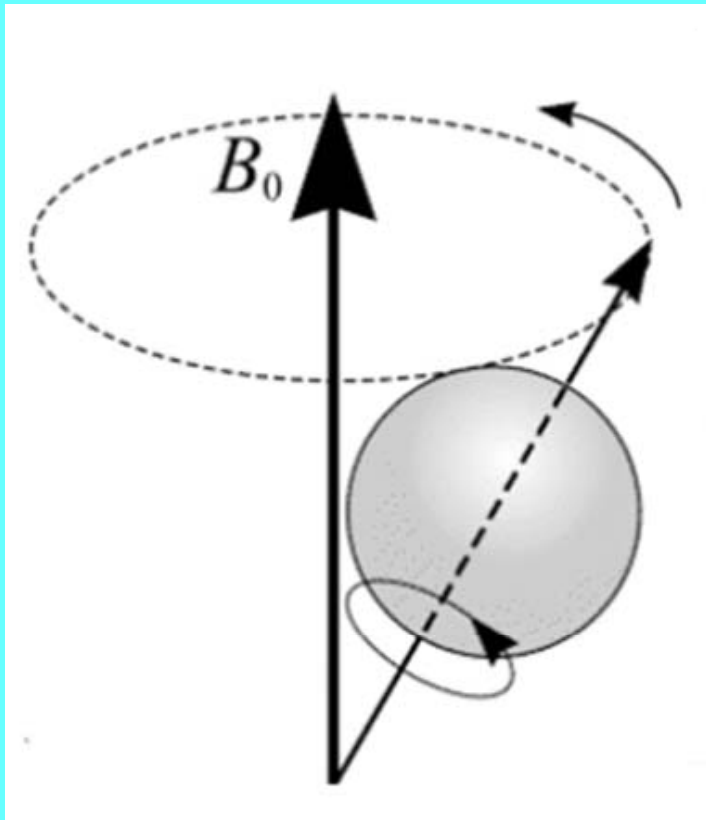
$$\Delta B = 1/\gamma T_1$$

Resonance field/frequency

$$\Delta g = g - g_0, \quad g_0 = 2.0023$$

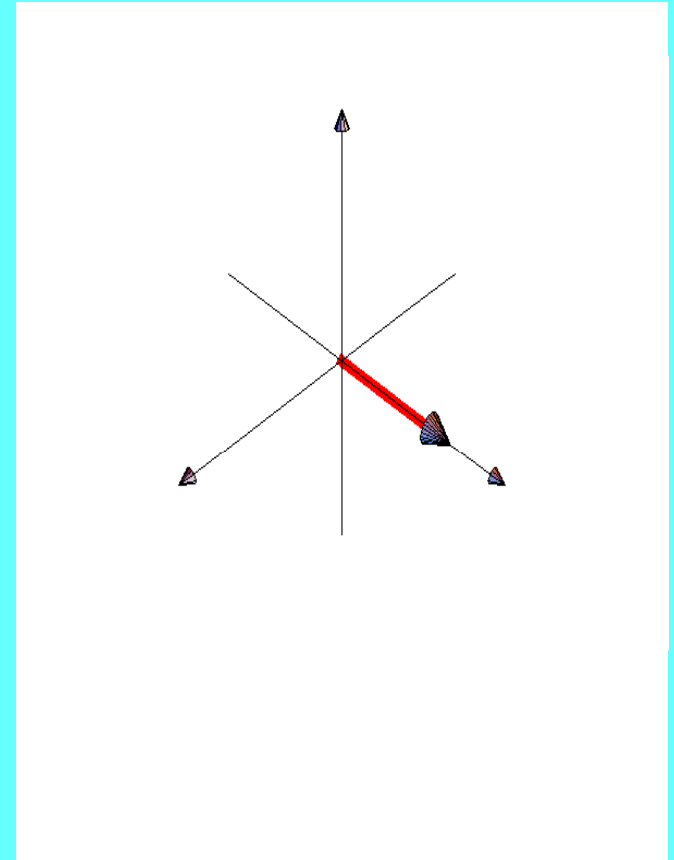


Phenomenology, Bloch Equations



Larmor-
precession
 $\omega_L = \gamma B_0$

B_1 : small in-
plane field



$$\frac{dM_{x,y}}{dt} = \gamma[\boldsymbol{\mu} \times \mathbf{B}_0]_{x,y} - \frac{M_{x,y}}{T_2}$$
$$\frac{dM_z}{dt} = \gamma[\boldsymbol{\mu} \times \mathbf{B}_0]_z + \frac{M_0 - M_z}{T_1}$$

T_2 : Spin-spin Rel.time

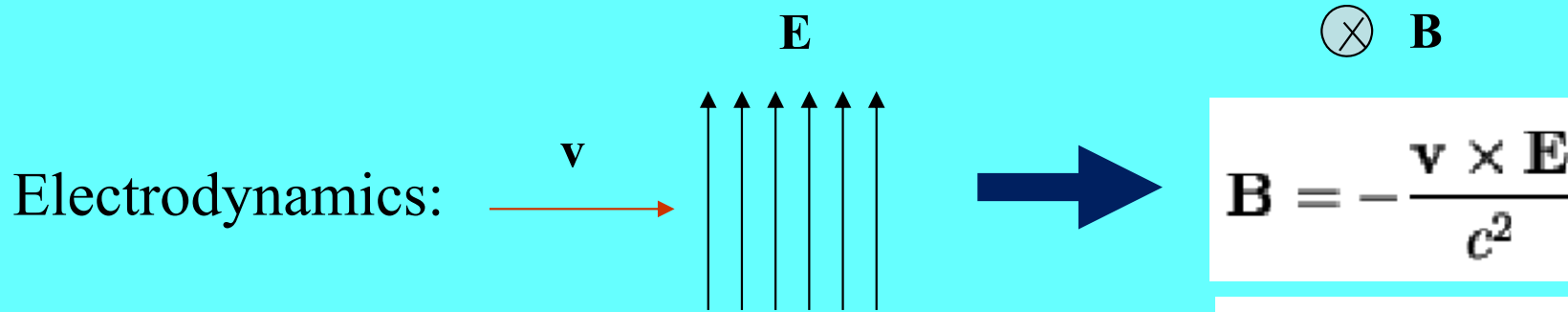
T_1 : Spin-lattice Rel.time

What is relaxation time?

	Bloch notation	Historic name	Modern name	Physical origin
T_1	Longitudinal	spin-lattice	Spin-relaxation	Energy transfer/ Fluctuating SOC
T_2	Transversal	spin-spin	Decoherence Irreversible decoherence	Like-spin change/ Motional narrowing+ Scattering/ Fluctuating SOC
T_2^*	non-Gaussian	Inhomogeneous broadening	Dephasing Reversible decoherence	Inhomogeneities, defects

Hierarchy: $T_2^* < T_2 \leq T_1$ in isotropic systems: $T_1 = T_2 = \tau_s$

Spin-orbit coupling



Interacts with electrons' magnetic moment

$$\hat{\mu} = -\frac{e}{2m}g\hat{\mathbf{S}}, \quad g = 2$$

$$-\mathbf{v} \times \mathbf{E} = \mathbf{v} \times \nabla \frac{Ze}{4\pi\epsilon_0 r} = \mathbf{v} \times \frac{\mathbf{r}}{r} \frac{d}{dr} \frac{Ze}{4\pi\epsilon_0 r} = \frac{1}{m} \hat{\mathbf{L}} \frac{Ze}{4\pi\epsilon_0 r^3}$$

$$\hat{H}_{\text{SO}} = -\hat{\mu}\mathbf{B} = \frac{Ze^2}{4\pi\epsilon_0} \frac{1}{2m^2c^2} \frac{\hat{\mathbf{S}}\hat{\mathbf{L}}}{r^3}$$

Types of internal E field:

Intrinsic (atomic)

Dresselhaus

Bychkov-Rashba

Proximity

e.g. GaAs

gating field

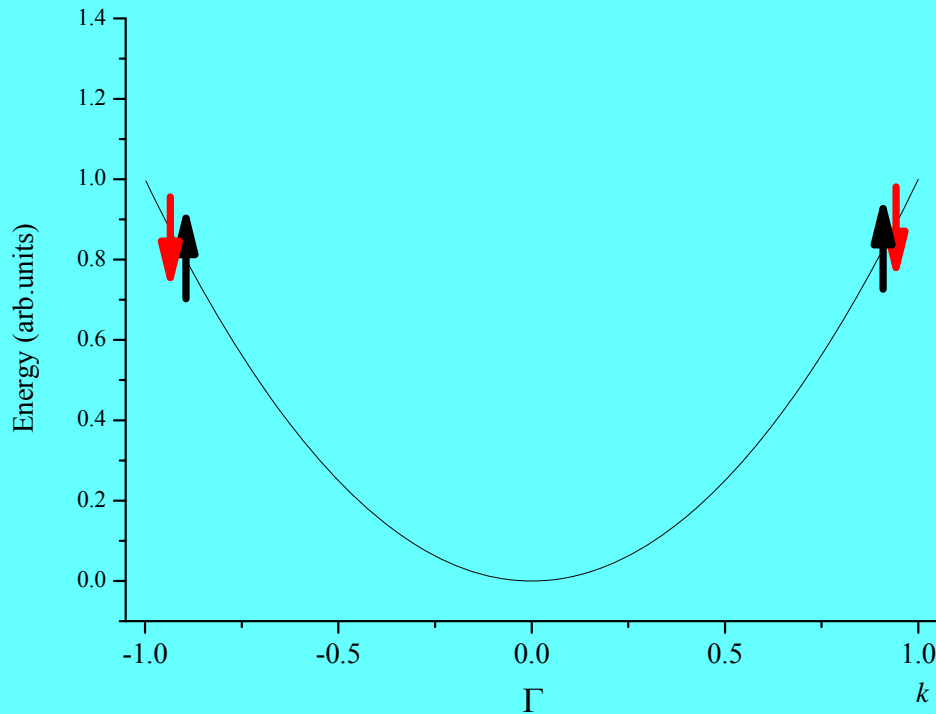
heterolayers

$$E_{n,l+\frac{1}{2}} - E_{n,l-\frac{1}{2}} = \frac{mc^2}{2} \frac{\alpha^4 Z^4}{n^3 l(l+1)}$$

Role of inversion symmetry

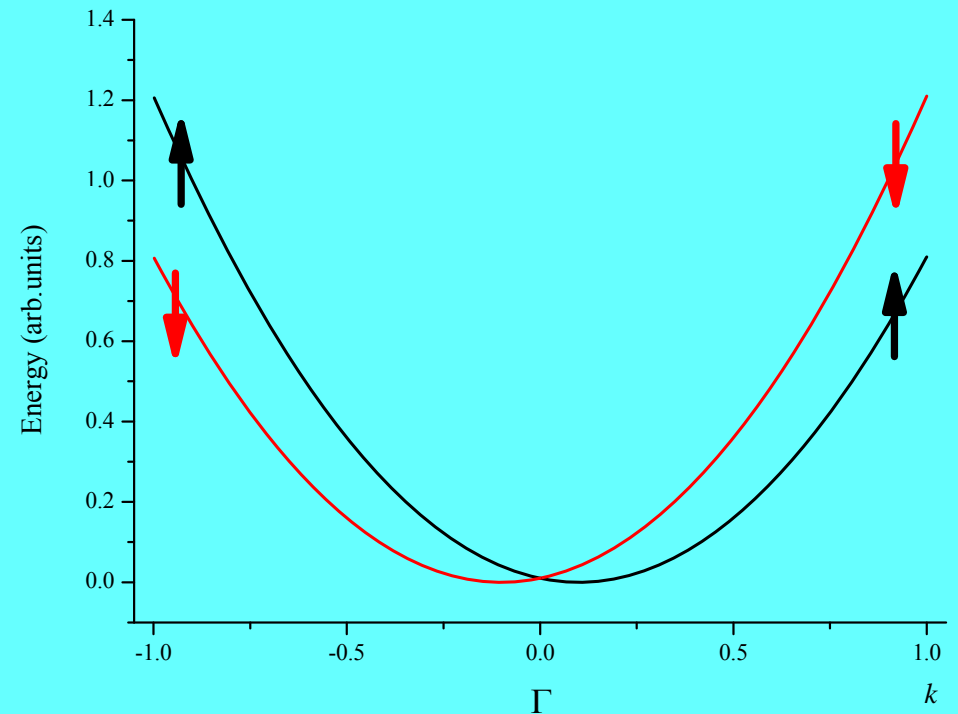
Time reversal: k, \uparrow and $-k, \downarrow$ are always degenerate even with SOC

inversion



k, \uparrow and k, \downarrow are degenerate

broken (GaAs)



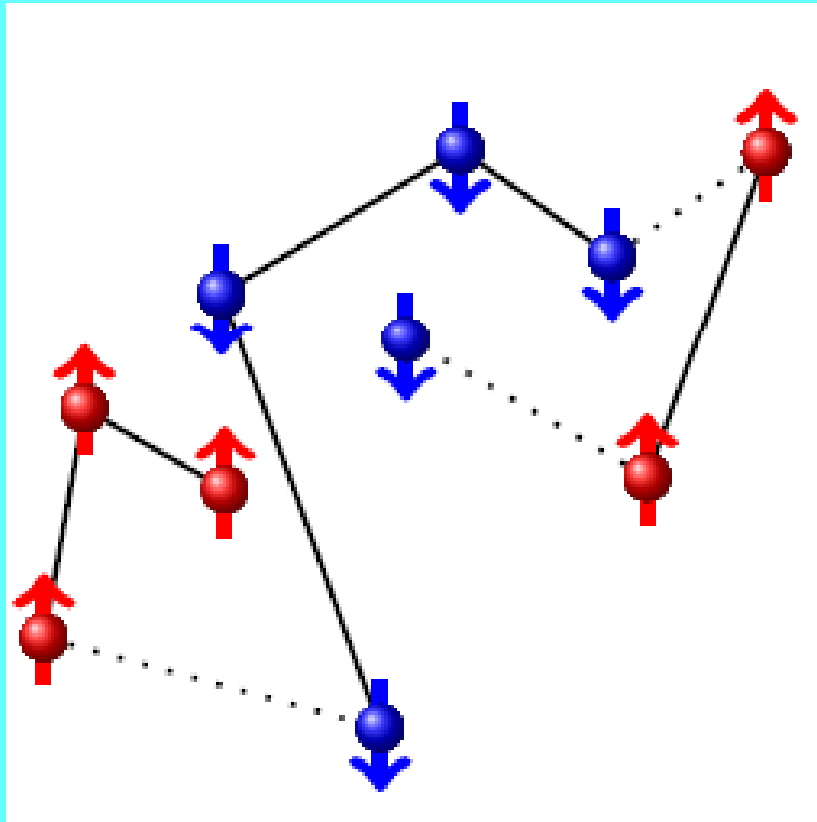
**Effective, k -dep.
internal magnetic field**

Phenomenology of spin-relaxation

inversion

$$1/\tau_s \propto 1/\tau$$

$$\Gamma_s \propto \Gamma$$



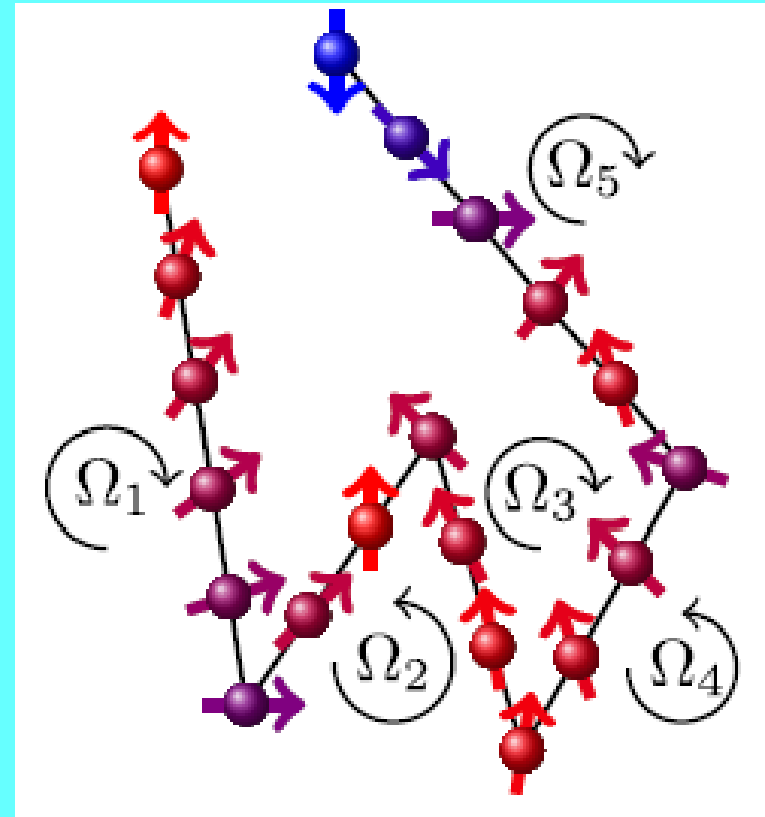
Elliott-Yafet

1st order pert. theory

broken

$$1/\tau_s \propto \tau$$

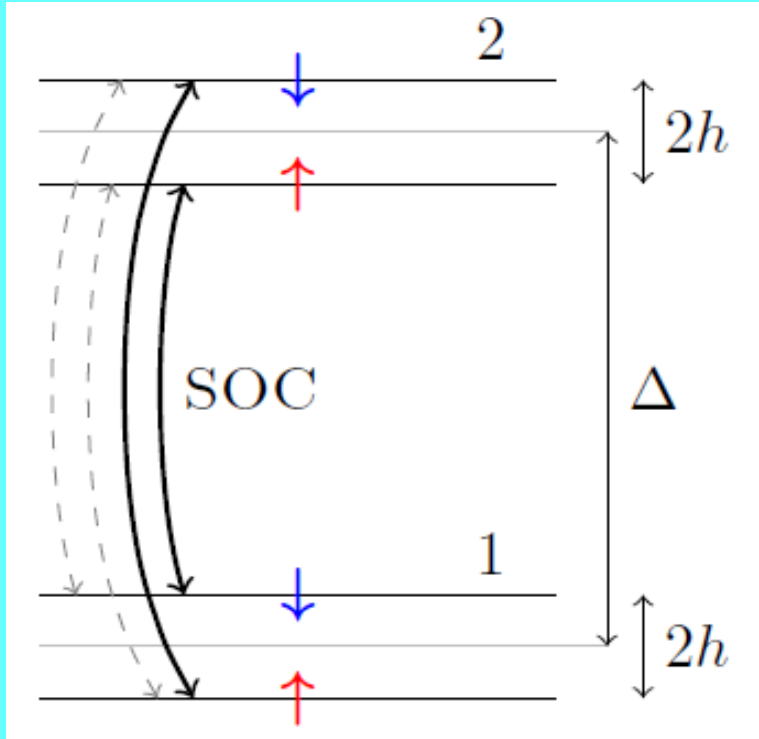
$$\Gamma_s \propto 1/\Gamma$$



Dy'akonov Perel'

“motional narrowing”

The Elliott-Yafet theory: Without SOC pure spin up/down states



$$\hat{\mathcal{H}} = \begin{pmatrix} h & 0 & 0 & L_{\mathbf{k}} \\ 0 & -h & L_{\mathbf{k}}^* & 0 \\ 0 & L_{\mathbf{k}} & \Delta + h & 0 \\ L_{\mathbf{k}}^* & 0 & 0 & \Delta - h \end{pmatrix}$$

$$|\tilde{\uparrow}\rangle_{\mathbf{k}} = [a_{\mathbf{k}}(\mathbf{r}) |+\rangle + b_{\mathbf{k}}(\mathbf{r}) |-\rangle] e^{i\mathbf{k}\mathbf{r}}$$

$$\frac{|b_{\mathbf{k}}|}{|a_{\mathbf{k}}|} \propto \frac{L}{\Delta E}$$

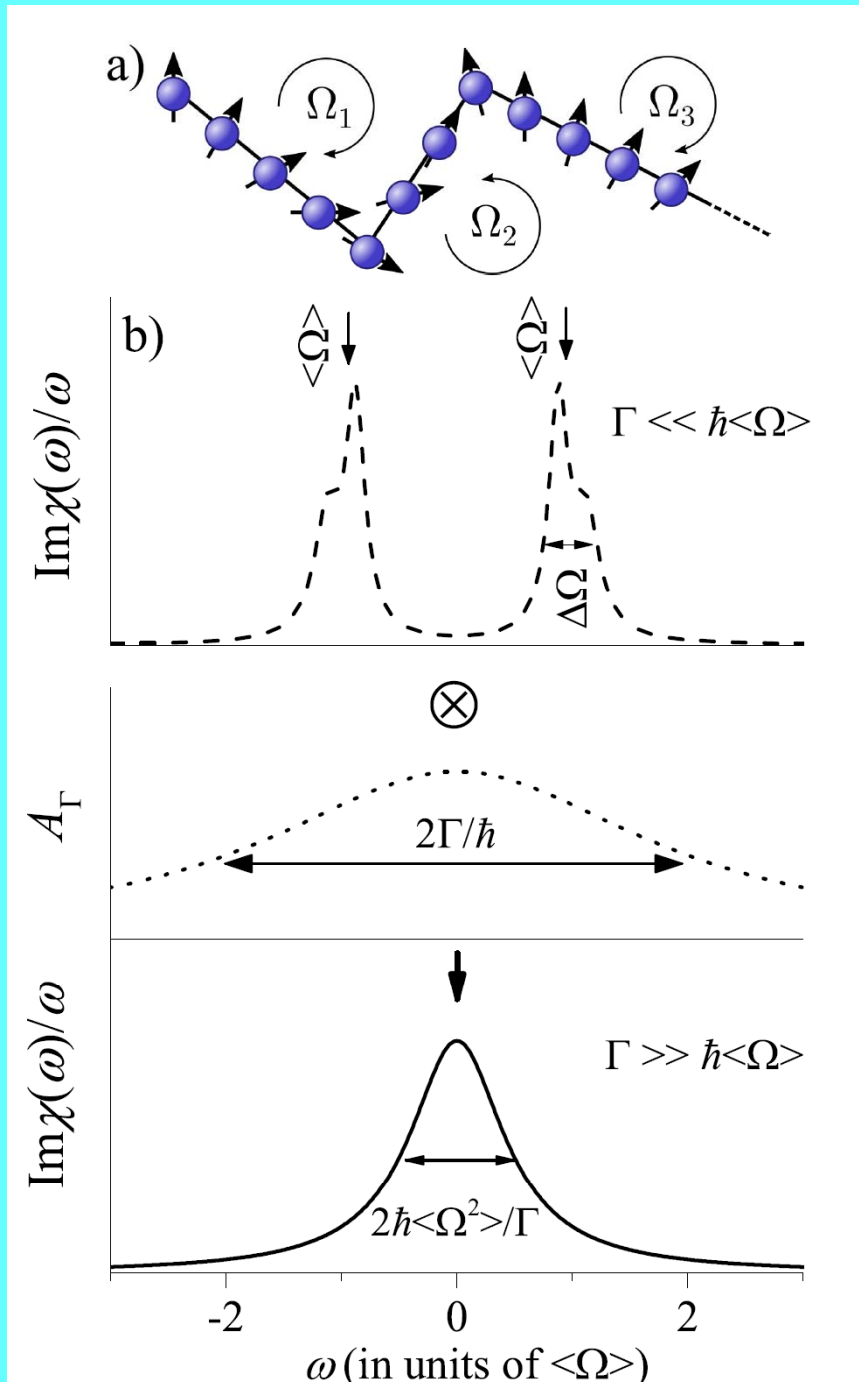
SOC mixes spin up/down states

Time dependent perturbation theory:

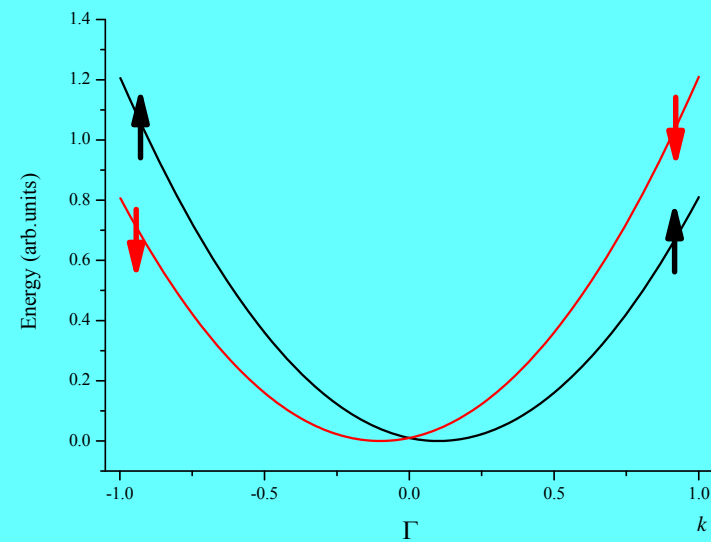
$$\Gamma_s = \alpha \frac{L^2}{\Delta^2} \Gamma$$

Proportional resistivity and ESR width!

The Dy'akonov-Perel' theory: internal fields



$$\text{SOC } (\mathcal{L}) \rightarrow \varepsilon(\mathbf{k}) \rightarrow \mathbf{B}(\mathbf{k}) \rightarrow \Omega(\mathbf{k})$$



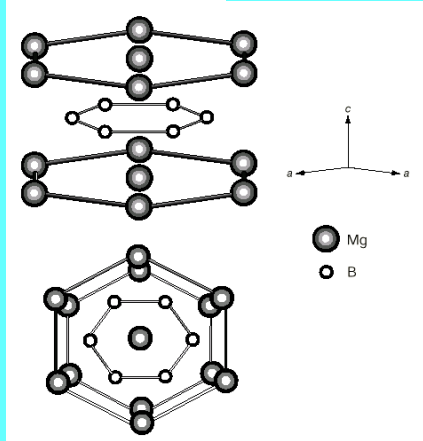
Condition: $\langle\Omega\rangle\tau \ll 1$

or

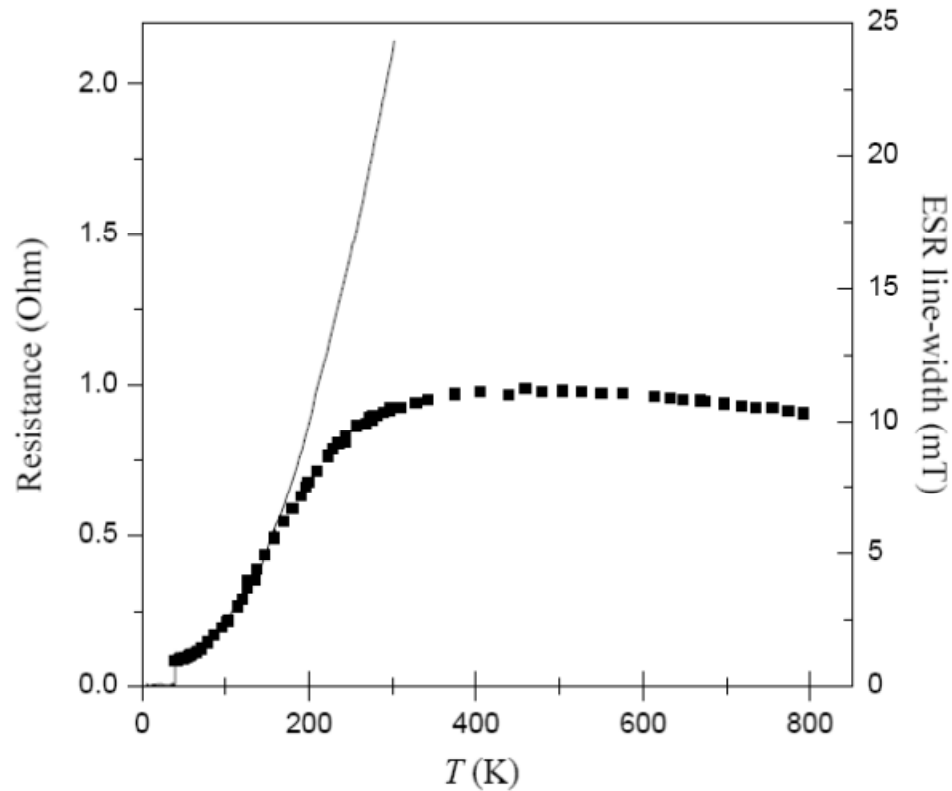
$$\mathcal{L} \ll \Gamma$$

Result:

$$\Gamma_s = \alpha \frac{\mathcal{L}^2}{\Gamma}$$



Anomalous spin-lattice relaxation (or line-width) in MgB₂



Anomaly appears above 150 K

No magnetic field-

No thermal history dependence

No purity, no isotope,
No sample type dependence

It is a true electronic effect

Reproduced by

Rettori et al. 2001

Monod et al. 2001

F. Simon et al. PRL 87, 047002 (2001).

The generalized Elliott-Yafet theory

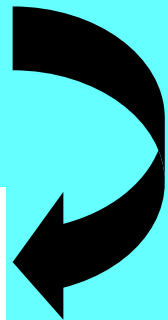
FS et al. PRL 101, 177003 (2008).

In EY: τ does not play a role, treated to lowest order

For elemental metals $\Delta \approx 10$ eV $\Gamma = 2\pi k_B T \lambda \approx 6$ meV

$\lambda = 0.1$ electron-phonon coupling at $T = 100$ K

$$\Gamma_{\text{spin}} = \left\langle \frac{L^2}{\Delta^2} \Gamma \right\rangle_{\text{FS}}$$

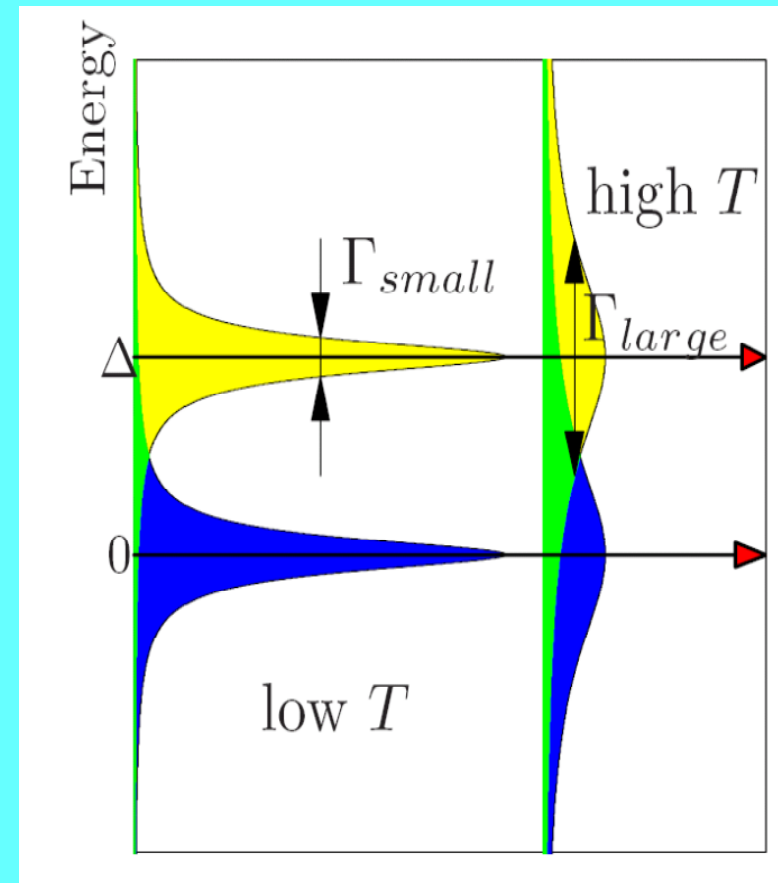


$$\Gamma_{\text{spin}} = \left\langle \frac{L^2}{\Delta^2 + \Gamma^2} \Gamma \right\rangle_{\text{FS}}$$

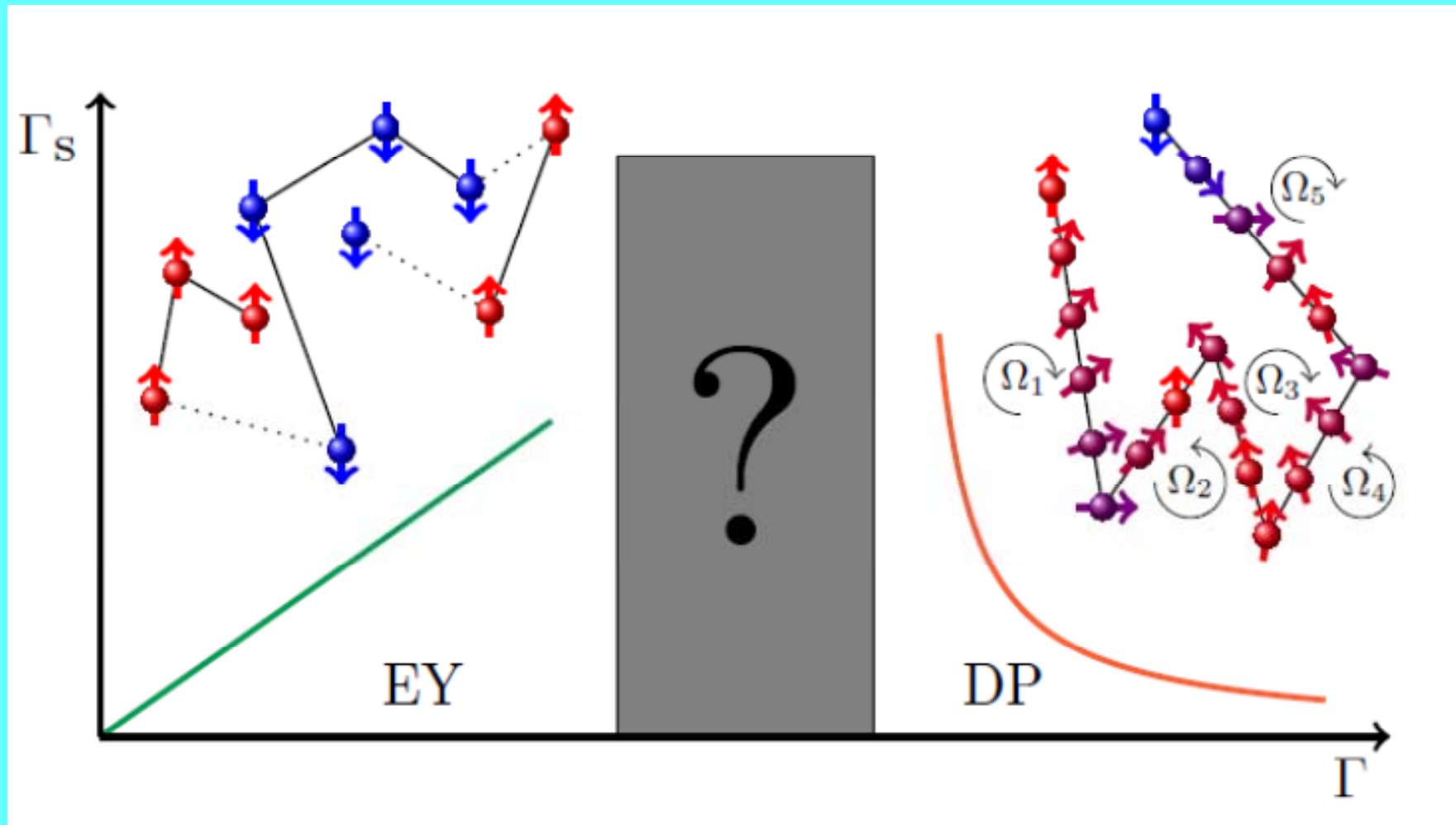
400 K-en, $\Gamma = 0.24$ eV

Large EPC!

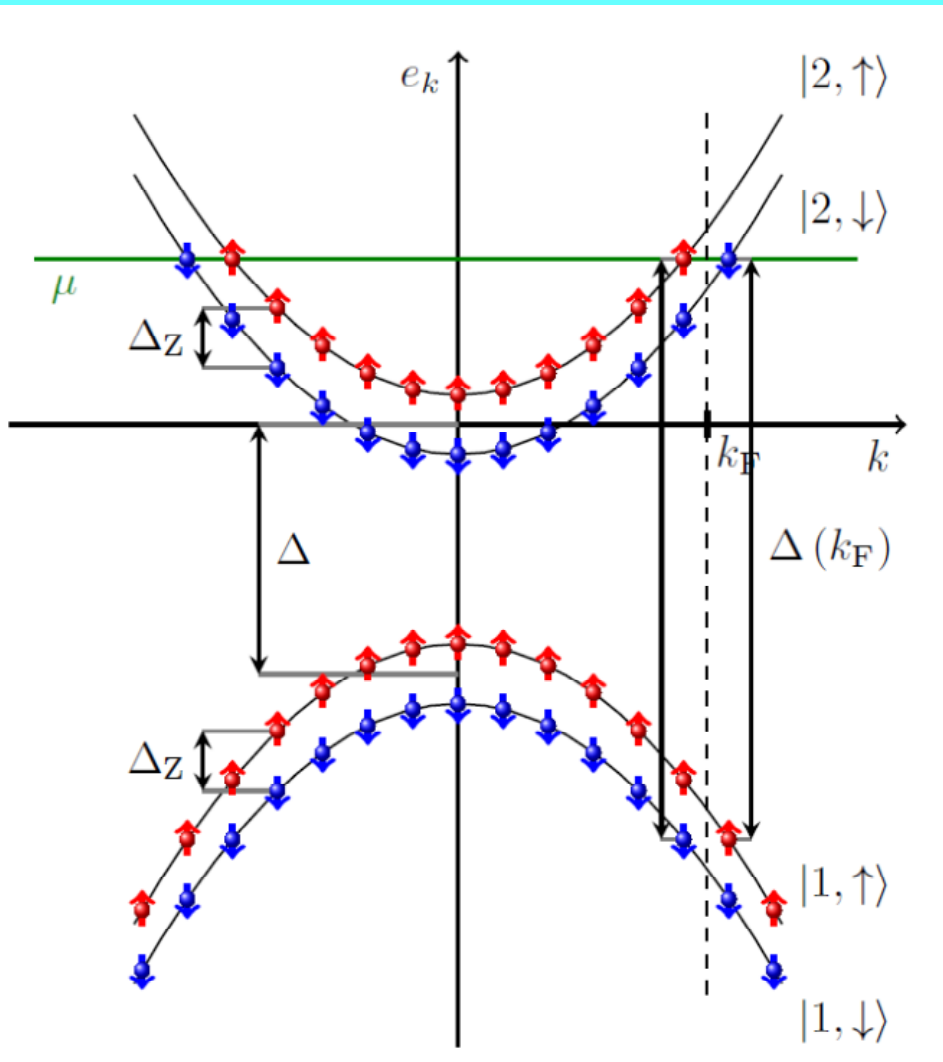
Where is the small gap?



The generalized treatment of EY and DP



The model: 2 DEG



$$L_{\alpha, \alpha', s, s'}(k) = \begin{pmatrix} 0 & \mathcal{L} & 0 & L \\ \mathcal{L}^\dagger & 0 & L^\dagger & 0 \\ 0 & L & 0 & \mathcal{L} \\ L^\dagger & 0 & \mathcal{L}^\dagger & 0 \end{pmatrix}$$

L: interband matrix elements
 \mathcal{L} : intraband matrix elements

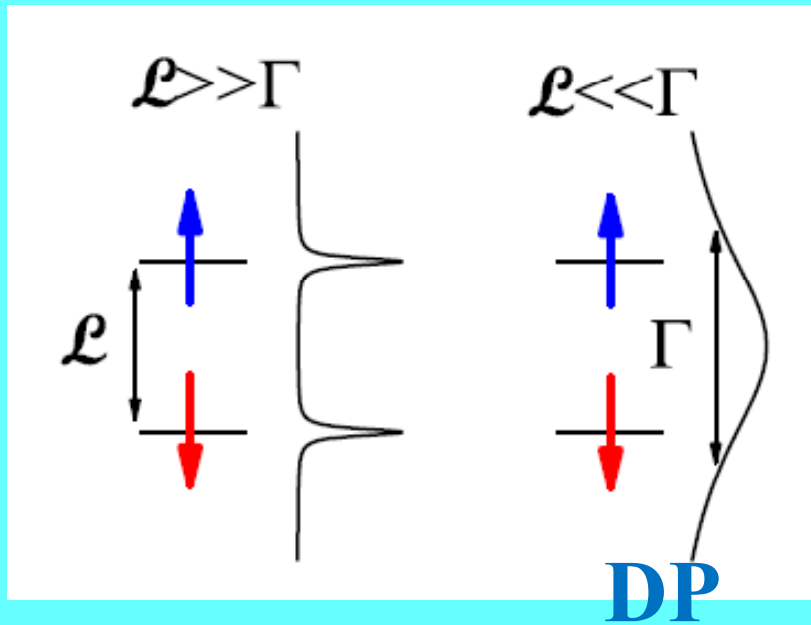
Dynamic spin susceptibility:

$$I(\omega) = \frac{B_\perp^2 \omega}{2\mu_0} \chi''_\perp(q=0, \omega) V$$

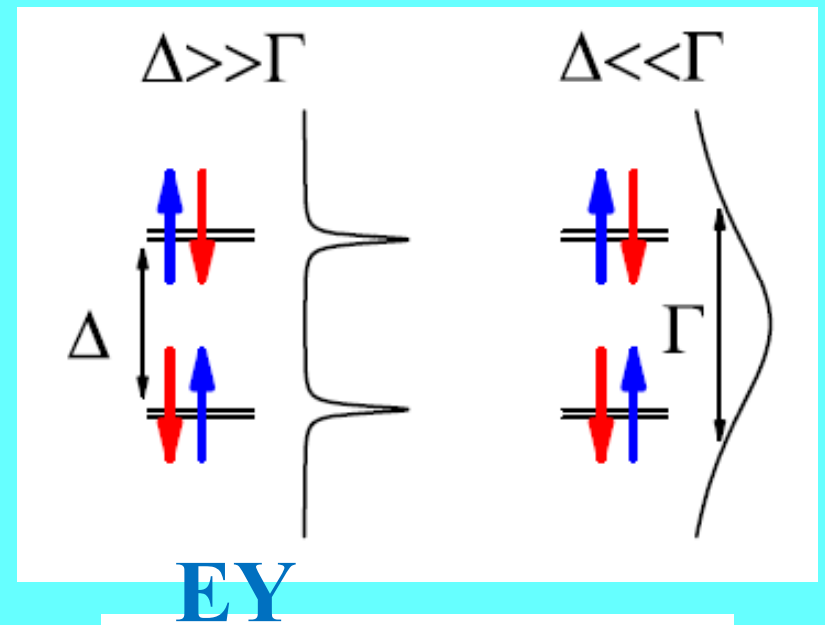
$$\Gamma_s = \frac{4\Gamma |\mathcal{L}(k_F)|^2}{4\Gamma^2 + \Delta_Z^2} + \frac{4\Gamma |L(k_F)|^2}{4\Gamma^2 + \Delta^2(k_F)}$$

Boross, Dóra, Kiss, Simon,
 Sci. Rep. 3, 3233 (2013).

The intuitive unification I.



$$H_{\text{DP}} = \begin{bmatrix} 0 & \mathcal{L}_k \\ \mathcal{L}_k^* & 0 \end{bmatrix}$$



$$H_{\text{EY}} = \begin{matrix} & \begin{matrix} 1\uparrow & 1\downarrow & 2\uparrow & 2\downarrow \end{matrix} \\ \begin{matrix} 1\uparrow \\ 1\downarrow \\ 2\uparrow \\ 2\downarrow \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & L_k \\ 0 & 0 & L_k^* & 0 \\ 0 & L_k & \Delta_k & 0 \\ L_k^* & 0 & 0 & \Delta_k \end{bmatrix} \end{matrix}$$

Add 2 virtual states+magnetic field

$$H_{\text{DP,amended}} = \begin{bmatrix} 0 & \mathcal{L}_k & 0 & 0 \\ \mathcal{L}_k^* & \Delta_{Z,k} & 0 & 0 \\ 0 & 0 & \Delta_{Z,k} & \mathcal{L}_k \\ 0 & 0 & \mathcal{L}_k^* & 0 \end{bmatrix}$$

Transform the states

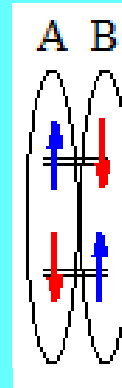
$$H_{\text{EY,renamed}} = \begin{matrix} & \begin{matrix} A\uparrow & A\downarrow & B\uparrow & B\downarrow \end{matrix} \\ \begin{matrix} A\uparrow \\ A\downarrow \\ B\uparrow \\ B\downarrow \end{matrix} & \begin{bmatrix} 0 & L_k & 0 & 0 \\ L_k^* & \Delta_k & 0 & 0 \\ 0 & 0 & \Delta_k & L_k \\ 0 & 0 & L_k^* & 0 \end{bmatrix} \end{matrix}$$

The intuitive unification II.

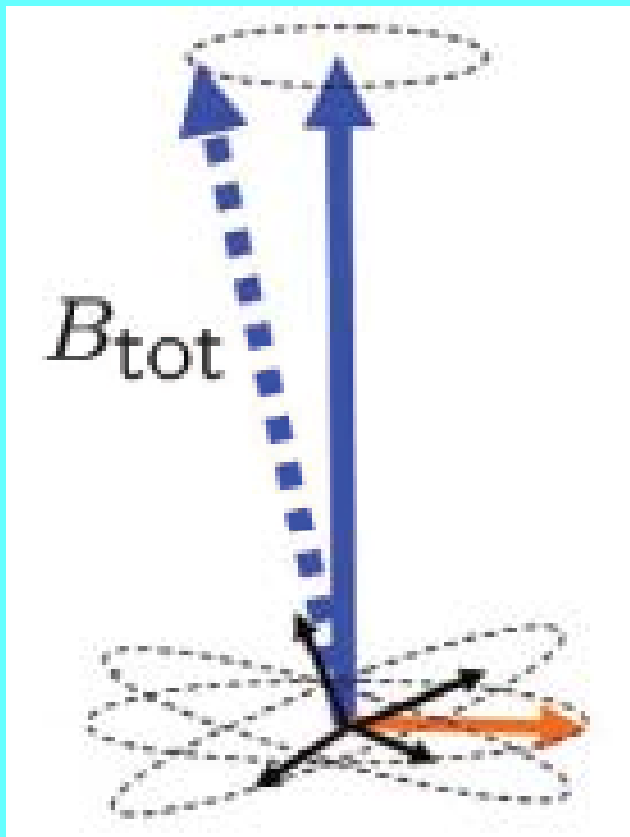
How to obtain the EY result in the DP language?

$$H_{\text{EY}} = \begin{matrix} & \begin{matrix} 1\uparrow & 1\downarrow & 2\uparrow & 2\downarrow \end{matrix} \\ \begin{matrix} 1\uparrow \\ 1\downarrow \\ 2\uparrow \\ 2\downarrow \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & L_k \\ 0 & 0 & L_k^* & 0 \\ 0 & L_k & \Delta_k & 0 \\ L_k^* & 0 & 0 & \Delta_k \end{bmatrix} \end{matrix}$$

$$\begin{aligned} |A \uparrow\rangle &= |1 \uparrow\rangle \\ |A \downarrow\rangle &= |2 \downarrow\rangle \\ |B \uparrow\rangle &= |2 \uparrow\rangle \\ |B \downarrow\rangle &= |1 \downarrow\rangle \end{aligned}$$



$$H_{\text{EY,renamed}} = \begin{matrix} & \begin{matrix} A\uparrow & A\downarrow & B\uparrow & B\downarrow \end{matrix} \\ \begin{matrix} A\uparrow \\ A\downarrow \\ B\uparrow \\ B\downarrow \end{matrix} & \begin{bmatrix} 0 & L_k & 0 & 0 \\ L_k^* & \Delta_k & 0 & 0 \\ 0 & 0 & \Delta_k & L_k \\ 0 & 0 & L_k^* & 0 \end{bmatrix} \end{matrix}$$



$$\frac{1}{T_1} = (\overline{\omega_x^2} + \overline{\omega_y^2}) \frac{\tau_c}{\omega_0^2 \tau_c^2 + 1} \quad (\text{IV.36})$$

J. Fabian *et al.* Acta Physica Slovaca
2007

$$\Gamma_s = \alpha \frac{L^2}{\Delta^2 + \Gamma^2} \Gamma$$

Same as the quantum kinetics approach!

A Monte Carlo approach to Spin-Relaxation



A Monte Carlo approach to spin-relaxation

Szolnoki, Sci. Rep. 2017

The model:

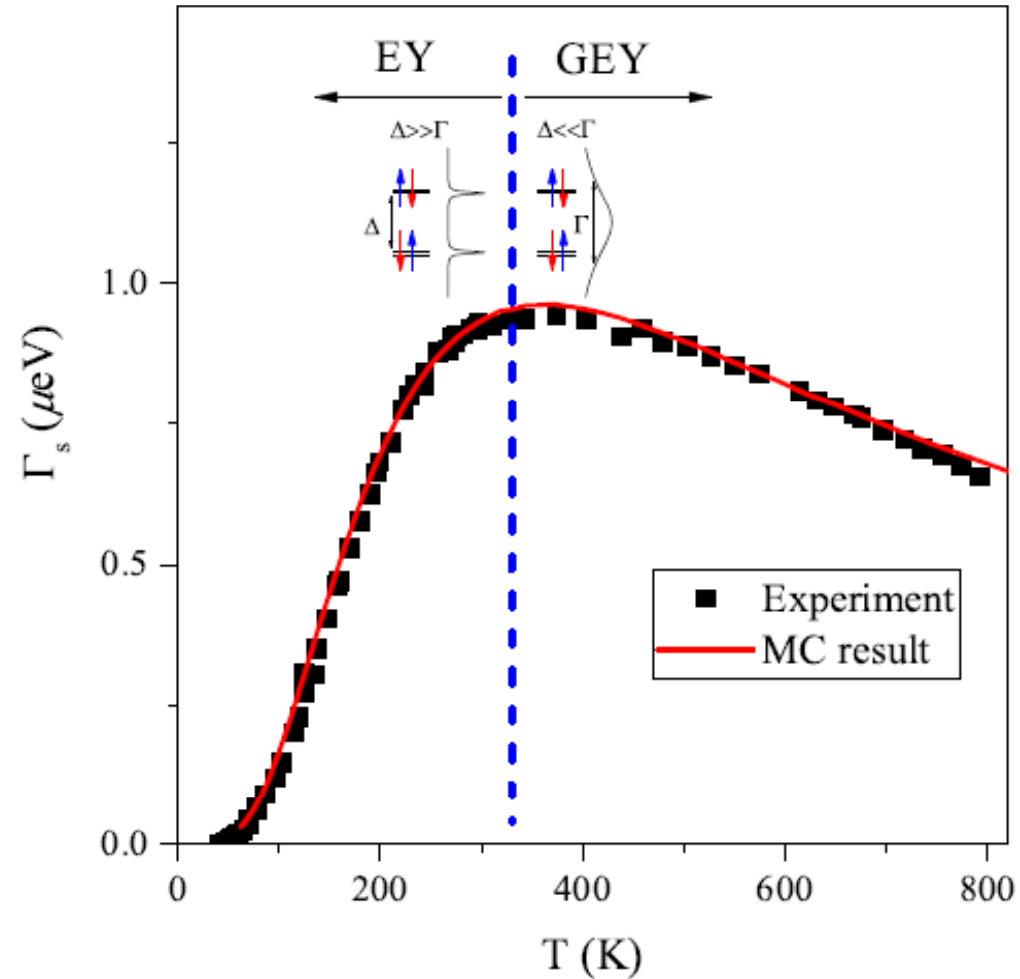
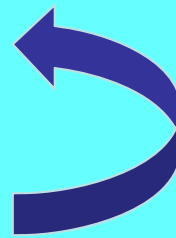
1. Polarized spin ensemble



2. Evolution due to $\Omega(k)$



3. New random k



Conclusions

Forget the simple classifying phenomenology of EY and DP

Think about EY spin-flip as continuous precession

Think about DP as a half of an inversionally symmetric system

The MC is useful to tackle SR for both DP and EY for arbitrary Γ

Coworkers:

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